Teaching Times Tables: A Whole School Approach

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In November 2017, a group of primary teachers from across the Tees Valley formed a Teacher Research Group for Archimedes Maths Hub which looked into the teaching of times tables. The project title was: 'How do we learn times tables? Is chanting enough?' Our findings suggested that rote learning was not enough to help children to recall or apply their tables facts and that a strategy-based approach was required to allow children to understand and make use of the properties of and connections within maths. The work led us to write this Programme of Study which gives a basic teaching order, with a suggested focus for each year group.

One of our key findings was that there are key addition and subtraction skills upon which multiplication strategies rely and, without these, children struggle to access and use multiplication facts flexibly. We therefore included these are part of the programme, in the hope that their importance for using tables facts flexibly will be recognised.

Our work is built upon the ideas of the Mastery Approach (NCETM, 2014) and activities begin by making explicit the structure and relationships within the maths through the use of practical equipment for a short period of time until the concept is understood. Visual representations are used alongside these and act as a bridge between the concrete and abstract. When concrete materials are removed, the visual representations can be used to allow children to continue to explore relationships and properties by allowing them to visualise and internalise connections within their work. When the concept has been fully understood, visuals can be removed and children can work completely in the abstract, although at times they may return to a visual representation to clarify their thinking or make the maths within a problem more accessible. We deliberately kept many of the visuals the same each year, with only the numbers changing, so that children can focus upon and understand that the relationships between numbers remain the same even when the numbers change.

We very much hope that our ideas for the teaching of key strategies in Key Stage One and Key Stage Two will provide schools with some useful ideas to support children in developing rapid recall of times tables through understanding the underlying structure of and relationships within multiplication.

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Kathryn Beaumont, Ormesby Primary; Alix Tripp, Nunthorpe Primary; Oriol Tyerman, Normanby Primary; Tanya Brown, Preston Primary; Emma Sutton and Laura Morley, St Teresa's Primary; Hannah Richardson, Corpus Christi Primary; Rebecca Evans, Skelton Primary; Kerry Mallam, Sacred Heart Primary; Judy Montier, Oxbridge Primary; Alex Herd and Susan Archer, Zetland Primary; Heather Clayton, Abby Hodgson and Laura Calvert, Fairfield Primary.

<u>Programme of Study written in collaboration with:</u>

Kathryn Beaumont, Ormesby Primary; Alix Tripp, Nunthorpe Primary; Tanya Brown and Claire Gence, Preston Primary; Emma Sutton and Laura Morley, St Teresa's Primary; Hannah Richardson, Corpus Christi Primary; Rebecca Evans, Skelton Primary; Kerry Mallam and Sharon Spillane, Sacred Heart Primary.

Contents

Introduction

The main aim of our group was to look at teaching times tables in a way that supports both recall and application. We all know that many children struggle to retain tables facts and can often feel overwhelmed by the sheer volume of facts to recall. However, when teachers are asked to talk about teaching times tables, conversation often ends up revolving around testing times tables, as though teaching and testing are the same thing. The difficulty with using testing as a means of improving recall is that, although it may motivate those who can already rote learn to increase their speed and confidence, it does not support those children who struggle with rote learning in making the connections needed to access tables facts. In fact, experience shows that it can often lead them to feel even more overwhelmed.

Research has found that maths facts are held in the working memory section of the brain (Beilock, 2011; Ramirez et al, 2013, cited in Boaler, 2015). Those with working memory difficulties are likely to struggle with memorising lists of tables facts. For them to access tables facts rapidly, they need to understand the relationships between numbers and work flexibly with these, so they can transfer knowledge to long term memory. Because working memory can become blocked in stressful conditions (ibid), even those who can rote learn may not be able to retrieve facts under pressure. Our own experience brought up the issue that rote learning can encourage children to see tables facts as isolated pieces of information which, for some, prevents them from making use of the relationships within them. A typical example of this is the child who is unable to work out 13x7 because they don't know their thirteen times table.

We decided that there seemed to be a strong case for developing a strategy-based approach to the teaching of times tables, which would build understanding by focusing on connections in order to support retrieval and application of knowledge. To align with the Five Big Ideas of Teaching for Mastery (NCETM, 2017), we looked particularly at which concrete and visual representations and structures supported this learning. This seemed to be particularly important as research has shown that work on mathematics involves five pathways in the brain and two of these are visual (Boaler, Munson and Williams, 2017). Strategies that allowed children to visualise number relationships and connections were incorporated wherever possible. Our work has resulted in this Programme of Study, which we hope will give teachers guidance in developing a structured approach to the teaching of times tables.

The Research: Obstacles and Solutions

Firstly, we researched successful times tables strategies and took them back to our schools to trial. However, we found that in all the schools there were a number of key obstacles to overcome in order for these to be successful. We have summarised the key findings and the suggested solutions that we believe are needed.

Obstacle:

- Over-reliance on skip counting. Many children, particularly those who struggle with recall, over-rely on skip counting and this makes it difficult to access tables facts when not in order. It also means they often fail to recognise or make use of the commutative aspect of multiplication (2x3=3x2) and instead use time-consuming, inefficient methods to access tables facts.

Solution:

- ✓ Give children multiple ways to access and apply tables facts by exploring different relationships early on, so skip counting becomes one aspect of multiplication rather than the primary strategy. When introducing concepts and strategies, use practical equipment and represent strategies visually to help children make connections and visualise relationships.

Obstacle:

- ❖ Key Stage Two children struggle to use flexible multiplication strategies because they do not have the required mental strategies in place from Key Stage One.

Solution:

- ✓ Ensure the underlying mental addition and subtraction strategies (see page 6) are recognised as an integral part of multiplication and given a high priority in Key Stage One. Maintain a strong focus on these throughout all subsequent year groups.

Obstacle:

- ❖ Children in Key Stage Two over-rely on formal written methods, instead of thinking logically about the numbers and concepts involved.

Solution:

- ✓ Explore a range of strategies from an early age; continue to develop mental methods which can be used for both arithmetic and reasoning questions throughout Key Stage Two, so children learn to make decisions and don't always default to the formal method.
- ✓ Delay teaching of formal written methods in Year Three until key mental methods have been explored and understood. (N.b. This is also advised in Year 3 objective).

Obstacle:

- ❖ Children feel overwhelmed by the number of tables facts to learn.

Solution:

- ✓ Make clear that only half the facts need to be learnt by exploring the commutative aspect of multiplication practically and visually. Ensure contexts for both scaling and skip counting (e.g. doubling in size as well as groups of 2) are used when each table is taught to avoid children gaining a one-dimensional view of multiplication as simply skip counting.
- ✓ Reduce the number of facts to be learnt in Year 4 by explicitly making links between the tenth multiple and x9, x11, x12 as part of the tables facts in Y2 and Y3, so that children already have an effective strategy for x9, x11,x12 with the tables they have covered.
- ✓ Revisit key strategies and relationships each year, using similar methods but with

different numbers, to allow cumulative learning where familiar structures are used to revisit prior learning, whilst at the same time supporting and scaffolding new learning. This supports working memory.
- ✓ Make use of the visual memory and known facts by using practical equipment and visuals to make links between fives and tens (Y2), twos and fours (Y3), fours and eights (Y3), threes and sixes (Y4).

Obstacle:

- ❖ Children struggle to apply tables knowledge within related contexts such as division and fractions.

Solution:

- ✓ Use visual structures and language which explore part whole relationships and make the inverse relationship between multiplication and division clear.
- ✓ Use arrays to share as well as group, so the link between sharing and grouping becomes obvious when dividing. Some children have difficulty understanding how counting in threes (grouping) results in finding a third (which is division as sharing). Sharing in an array can help clarify this understanding and support understanding of fractions. (See examples on page 12-15).

During the project, we found that, although a strategy-based approach worked for all children, it was more successful the earlier it was introduced as there were fewer obstacles to overcome.

Key Factors for Success

There are key skills upon which multiplication strategies rely. These must become automatic if strategies are to be used successfully to retrieve table facts at speed. They are:

- Partitioning.
- Doubling/ halving.
- Number bonds within ten/multiples of ten.
- Bridging.

A strong understanding of place value is also required in order for tables facts to be applied with larger numbers and decimals.

Our experience showed that lack of confidence with these underlying skills significantly impaired a child's ability to develop alternative strategies for accessing times tables facts. Therefore, these key skills must be given a high priority throughout Key Stage One and Two.

An important point to note is that the children who struggle to access and retain table facts are also likely to struggle to retain underlying skills and facts, yet they are fundamental to success. For this reason, we have also included strategies to support these underlying skills in Key Stage One and early Key Stage Two. Beyond this, opportunities to reinforce these skills are outlined.

Strategies For Use in Each Year Group

The Counting Stick

This is a wooden stick, one metre long, divided into ten equal parts. If you don't have one, a metre stick can easily be converted using tape to divide. An empty number line can also be used in the same way. This is a powerful way to represent number relationships and can also be used in Foundation Stage for counting in ones and finding missing numbers.

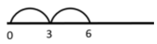
The example given is for 3x table but the same strategy applies regardless of the table being taught.

When first using with younger children, use sticky notes with the multiples already written on, gradually removing each multiple in the order given and explaining how we can work out the missing multiple even when it isn't there using the other multiples. After removing each multiple, count through the multiples again, forwards and backwards, including those which have been removed. As children become more confident, you could begin with empty counting stick and use the 10th multiple to work out the first.

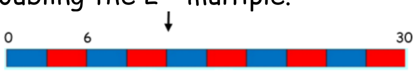
What is the first multiple? What table are we exploring?

Double the first multiple to find 2x

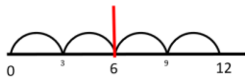
2x3



Discuss how the 4th multiple can be found by doubling the 2nd multiple.

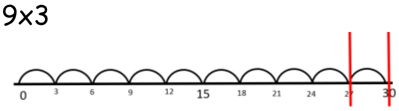


4x3

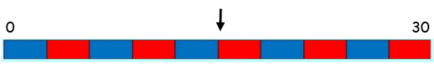


The 9th multiple can be found by subtracting one multiple from the 10th multiple, i.e. 30-3).

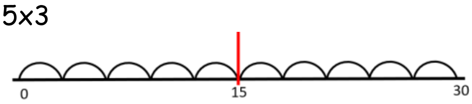
9x3



5th multiple – halve the tenth multiple).

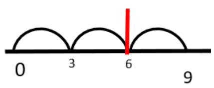


5x3



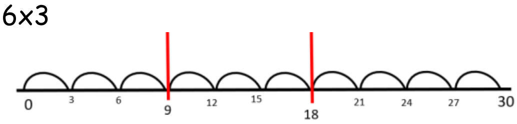
3rd multiple – triple the first multiple or double then add multiple.

3x3

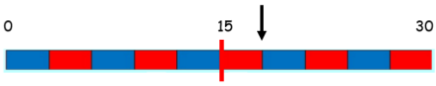


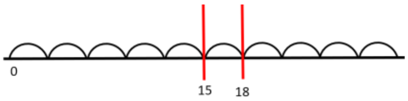
6th multiple – double the third multiple,

6x3



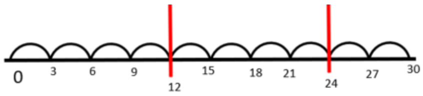
Or find 5x and add another multiple.



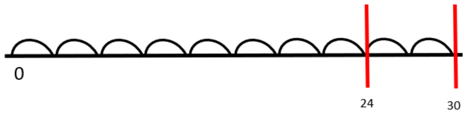


8th multiple - double the 4th multiple,

8x3

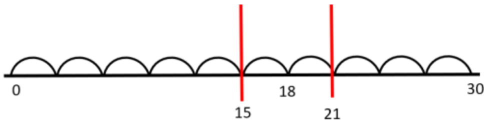


or remove the double from the 10th multiple.

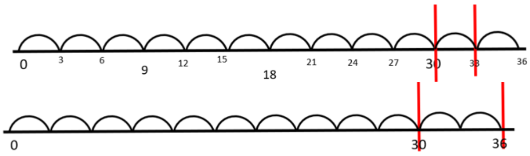


7th multiple – find 5x and add the double (2x).

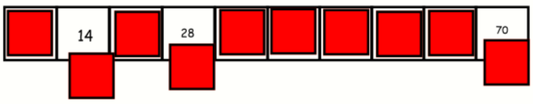
7x3



Discuss how 11x and 12x could be found.

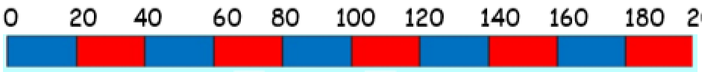


- Younger children may make their own number line by putting sticks of cubes of appropriate multiples along it and marking what number it would be to gain initial understanding of counting stick.
- Number cards with multiples on can be put face down onto an empty number track and a game played where an adult has to point to a card and the child predicts what it will be, explaining how they know. Begin with the easiest multiples first (e.g. first, second, ninth etc). The adult can use their turn to model language and the strategy used. E.g. I knew it (4th multiple) would be 28, because I knew 4x would be double 2x. This can be made as a homework game or played with the class on a number line.

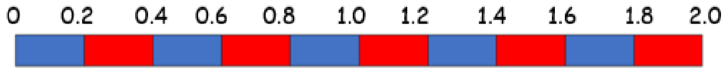


- Muddle up the multiples, or swap around two multiples, and ask the children to put them in the correct order, explaining why it was wrong in the first instance.
- Use the counting stick without any multiples on it, or only one, so the children have to use their knowledge or doubling, tripling, halving to derive the multiples of a given number. This can help with finding the multiples of larger 2 digit numbers for long division.
- Use alongside practical equipment (e.g. place value counters) to help develop understanding of place value with larger numbers and decimals. This relieves memory load by showing how the relationships stay the same, so the same strategies can be used.

E.g.



Or



The language used and questions asked can also show the link to division.
E.g. 28÷7=? Or how many sevens are there in 28?
I know there are 2 sevens in 14, so there must be 4 sevens in 28.

For a video example see: Times Tables in Ten Minutes. Available at: <https://www.youtube.com/watch?v=yXdHGBfoqfw.>.
Please note, this is not to show a method to memorise the 17x table, but a strategy to show how to use number relationships to find multiples of any of the times tables.

Using Multiplication Grids

With older children, multiplication grids can be rearranged to highlight the links between different tables, investigate patterns and help reduce memory load. Some examples are given below.

	1x	2x	3x
1	1	2	
2	2	4	
3	3	6	
4	4	8	
5	5	10	
6	6	12	
7	7	14	
8	8	16	
9	9	18	
10	10	20	

Fill in the tables grid for your 1 and 2 times table. Now fill it in for your 3x table. What do you notice? This strategy could be adapted to show the link between 10x, 5x and 15x.

Use the tens times table to help find your 5x facts.

	10x	5x
1	10	
2	20	
3	30	
4	40	
5	50	
6	60	
7	70	
8	80	
9	90	
10	100	

This could be adapted to explore multiplying by twenty or to explore the link between x1 and x0.5 (or x $\frac{1}{2}$) as well as other obvious doubling relationships (twos/ fours/ eights or threes /sixes/ twelves).

Fill in the tables grid for your 10 and 1 times table. Now fill it in for your 9x table.

	10x	1x	9x
1	10	1	
2	20	2	
3	30	3	
4	40	4	
5	50	5	
6	60	6	
7	70	7	
8	80	8	
9	90	9	
10	100	10	

What do you notice? Why does this happen? Can you prove it? Does it work with other numbers? Can you use this to find new facts?

Fill in the tables grid for your 10 and 2 times table. Now fill it in for your 8x table.

	10x	2x	8x
1	10	2	
2	20	4	
3	30	6	
4	40	8	
5	50	10	
6	60	12	
7	70	14	
8	80	16	
9	90	18	
10	100	20	

Dev says that if you know your doubles and your five times table, you can easily find your 7x facts. Is he right? Explore different numbers on a multiplication grid to find out. Explain what you find out.

Number to test	5x	Double (x2)	7x
3	15	6	21

It is important to focus upon making links to known facts and stressing that often one fact can help find another. Encourage children to identify those tables fact they find most difficult to remember and which are the easiest for them. Are there any ways they could use what they know already to help them find new facts?

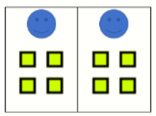
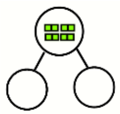
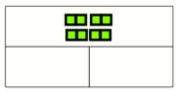
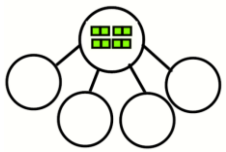
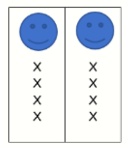
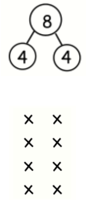
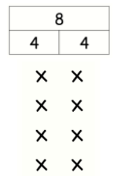
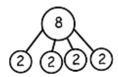
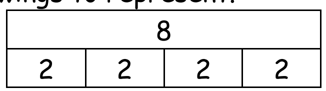
Rather than the traditional multiplication grid which shows 100 facts (for up to 10x10), it can be less daunting to make use of the commutative property and take out the facts that are already known from other tables. This reduces the number of facts to learn from 100 to 55. When the x1 and x10 facts are removed, only 36 are left. 26 of these are in the twos, threes, four or fives times table, so it is worth focusing on the doubling, halving and bridging skills required to find facts from these tables quickly. Providing a strong focus has been maintained on doubling, halving, children should be able to use these to quickly find the 7 facts still unknown from the six and eight times table. This leaves 9x7, 9x9 and 7x7 to learn and children should already have an efficient strategy for multiplying by 9. This only leaves 7x7. Breaking down tables facts in this way can help both the teacher and the children feel less overwhelmed!

1x1									
2x1	2x2								
3x1	3x2	3x3							
4x1	4x2	4x3	4x4						
5x1	5x2	5x3	5x4	5x5					
6x1	6x2	6x3	6x4	6x5	6x6				
7x1	7x2	7x3	7x4	7x5	7x6	7x7			
8x1	8x2	8x3	8x4	8x5	8x6	8x7	8x8		
9x1	9x2	9x3	9x4	9x5	9x6	9x7	9x8	9x9	
10x1	10x2	10x3	10x4	10x5	10x6	10x7	10x8	10x9	10x10

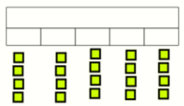
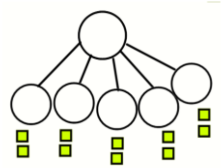
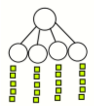
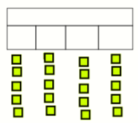
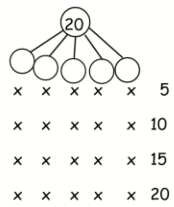
A grid similar the that above can be used as a starting point to discuss different strategies for finding tables fact. Eg. Point to 9x6 and ask the children to explain and show different ways to find that fact. Examples may be: Doubling 9x3, finding 10x6 and subtracting 6, finding 9x5 and adding 9 etc. Strategies can be shown on number lines, with arrays (or blank rectangles drawn to represent arrays with larger numbers) or bar models to help clarify thinking and to make explicit how the relationships and properties that exist within multiplication can be used to relieve memory load and find new facts.

Inverse Relationships

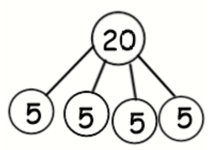
Year One

Notes	Concrete	Visual – moving towards Abstract
Children should work practically to divide by 2, 5 and 10 and draw what they have done in ways that are meaningful to them. Teacher can model using part whole models/bar models, talking about the parts and whole amounts and linking it to multiplication through the discussion. Discuss the idea of equality and give examples where groups are unequal to discuss.	Sharing: E.g. I have 8 stickers and I share them between my 2 friends. How many stickers do they get each? Share practically and then explore how it could be shown visually (drawing). Teacher can demonstrate how it would look in a part whole or bar model. Talk about what is the whole amount and what are the parts?    Grouping: Problems involving grouping should also be included so children can use practical apparatus to act out the problem. The bus had room for 2 people on each seat. If there were 8 children getting on the bus, how many seats would they fill?  Take off 2 children each time to fill a seat and teacher can draw part whole model as doing it. Discuss the whole amount and the parts. Relate it to everyday problems. Also, give examples solved incorrectly. E.g. Share some cubes out but give one person more than the others. Discuss if this is fair and relate to the idea of equal groups for both sharing and grouping.	Sharing: Encourage children to draw the children (stick people or smiley faces) and crosses to represent the objects. Teacher can model recording on part whole or bar models. E.g. 8 shared between 2.    When discussing sharing between 2, make the link to halving. Also, use bar models or part whole models to represent practical situations where sharing is between 5 or between 10. Grouping: Encourage children to work through the problem and draw, for example, squares to represent the seats on the bus, then put cubes on them. Eventually it can be recorded with crosses to represent the cubes.  Teacher can model on part whole or bar models but children may use their own drawings to represent.  

Year Two

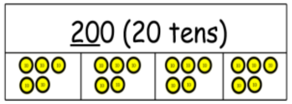
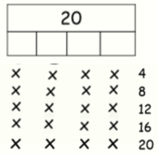
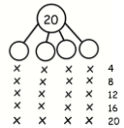
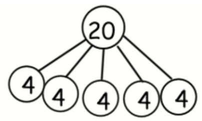
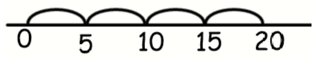
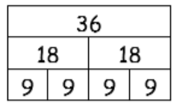
<table>
<tr><th>Notes</th><th>Concrete</th><th>Visual- moving towards Abstract</th></tr>
<tr>
<td>Explore the relationship between grouping and sharing as this can confuse children later and cause difficulty when solving division problems which requires sharing (e.g. fractions) but use multiples of a number (grouping).
Ensure equal grouping is understood.
Include division by 1 in both grouping and sharing contexts.
Work with dividing by 2,5 and 10.
Division by 3 and 4 is also required for fraction work on finding one third and one quarter.</td>
<td>Solve real life division problems practically using cubes and other apparatus to represent real life objects for both sharing and grouping.
Draw part whole models and bar models and relate practical work to these, discussing which are the parts and which is the whole.
Sharing: (20 shared between 5)
20÷5

Share out between 5.
10÷5

By sharing in an array, the link to grouping can be made. E.g. 20 must be shared out between 5 until it is all shared. Children can count each line of the array in fives to check the correct number has been shared to reinforce the tables link. This helps children see how counting in multiples links to division as sharing.
Grouping: (20 grouped into 5s)
20÷5

Group into fives and take five off the whole each time.
10÷5

Draw a bar underneath the whole each time 5 is taken off.</td>
<td>Children draw/complete part whole models or bar models themselves, using crosses to represent the objects shared or grouped if needed. (Numbers at the end of each line of an array can be counted orally, rather than recorded).
Sharing:
E.g. 20÷5


Arrays can also be useful for showing the link to fractions.
E.g. $\frac{1}{4}$ of 20

N.b. When sharing between 2, ensure the link to halving is understood. When finding quarters, bar models can also be used practically and visually to show how $\frac{1}{4}$ can be found be halving and halving again (see example on page 13).
Grouping:
Children may draw their own pictures to solve problems as in the Year One example, before recording on part-whole or bar models.


E.g. 20÷5
Take off a group of 5 each time until the whole has all been grouped.
These visuals can also be useful for exploring missing number problems E.g ___ ÷ 4=5 or 20 ÷ ___=___.
Children can also write their own problems to match visuals.</td>
</tr>
</table>

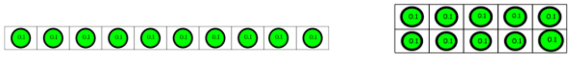
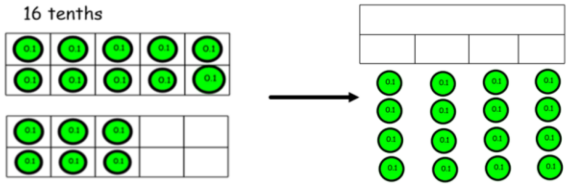
Years Three and Four

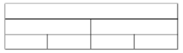
Notes	Concrete	Visual	Abstract
Use the same strategies as in Y2 to explore the sharing/ grouping link. Include division by 1 in both grouping and sharing contexts. These strategies can also be used for work on fractions. When working with larger numbers, e.g. 200, also talk about it as 20 tens (200) when sharing so children see the link to dividing 20 ones.	Sharing: When sharing practical apparatus as part of real life problems it can useful to share in an array on a part whole model or bar model. Count in multiples down the side or use them to count whilst sharing. Gradually move towards using multiples to solve, without concrete or visuals. Grouping: Divide by grouping practically by taking off groups of a given multiple using a range of representations to show the repeated subtraction. E.g. How many groups of 4 in 20? This can be solved practically using a part whole model, bar model or a number line. Use place value counters/Dienes to make links to known facts when solving sharing and grouping calculations with larger numbers.  E.g. Use bar/part whole model to solve 200÷ 4 or 20 tens÷4 Use to explore problems, including those with remainders, and write calculations for missing numbers e.g. __÷ 4=50. Find 100 ÷ 10, 200 ÷ 10 300 ÷ 10. What do you notice? Links between tables should be explored when dividing as well as multiplying. E.g. Explore practically and visually the link between dividing by 3 and dividing by 6 and link this to work on fractions .	Sharing: 20÷ 4 or $\frac{1}{4}$ of 20  Grouping: Grouping can also be be shown on a number line. 20÷4 20÷5  Talk about 20 tens, 20 hundreds, when working with larger numbers to support the link to ones. E.g. 2000 ÷ 4 is 2000 (20 hundreds) ÷ 4 etc. E.g. Use visuals to show: 12 ÷ 3. 12 ÷ 6. 18 ÷ 3. 18 ÷ 6. 24 ÷ 3. 24 ÷ 6. What do you notice? Show it on a bar model. Explore other numbers. What do you find out? E.g. Use concrete apparatus to show 24 ÷ 4. How can this help you find 24 ÷ 8? (Also explore the doubling/halving relationships with fractions).	Practical and visual representations can be used to solve missing number problems with division until the relationships are fully understood and these can be solved in the abstract. E.g. __÷ 4 =5. 30÷ __=5 What is the whole amount and what are the parts? Now draw the part whole model/bar model to find the missing number. Strategies for ÷4/finding a quarter (halving and halving again) or ÷8 (halving, halving and halving again) should be explored practically on bar models, before recording visually. E.g. 36 ÷4 or $\frac{1}{4}$ of 36.  Word problems can be written to match the visual or calculation and vice versa.

Years Five and Six

<table>
<tr><th>Notes</th><th>Concrete</th><th>Visual</th><th>Abstract</th></tr>
<tr>
<td>Explore the relationships between larger numbers and decimals to show that, although the place value has changed, the relationships stay the same.

The doubles and halves relationships within the tables should be exploited to help with division.

Include division by 1 and division by 10 regularly.</td>
<td>
Use strips or tens frames with 10 tenths shown (place value counters/Dienes tens) to represent 1. Use these to show the effect of dividing by 10 resulting in decimal numbers. E.g. 4÷ 10 is 40 tenths ÷ 10 which is 4 tenths. Ensure this is explored alongside the place value slider or grid and encourage children to visualise the number of tenths in whole numbers before calculating in the abstract.
E.g. 1.6÷4   1.6 or 16 tenths
Sharing:


Use Dienes or larger place value counters to show the same relationships with larger numbers.
E.g. Use plave value slider or grid alongside place value counters to show the effect of dividing by 1000, or to link known facts to work with larger numbers.
Investigate factor relationships.
E.g. True or false: 70÷ 20 can be found by dividing by 2 then dividing by 10. Prove it. Do you think you could divide by 10 then divide by 2? Explain.</td>
<td>Use the visual as a step between the concrete and the abstract and to explore relationships. E.g. 1.6÷4 or $\frac{1}{4}$ of 1.6 could be visualised on a bar model, thinking of 16 tenths.

Select 3 and 4 digit multiples of ten/hundred as a starting point to investigate patterns in division.

E.g. Predict the next three calculations. Explore how partitioning into 160+1, 160+2 etc can help with mental division.
160÷4=
161÷4=.
162÷4=
Factors should also be used to divide, focusing particularly on the relationship between ÷10 and ÷5 or ÷20 and exploiting doubles and halves relationships within the tables.
E.g. ÷20 by ÷10 then ÷2.
÷5 by ÷10 then doubling.
Relationships can be shown visually.
E.g. 80÷5
<table><tr><td colspan="10">80</td></tr><tr><td>8</td><td>8</td><td>8</td><td>8</td><td>8</td><td>8</td><td>8</td><td>8</td><td>8</td><td>8</td></tr><tr><td colspan="2">16</td><td colspan="2">16</td><td colspan="2">16</td><td colspan="2">16</td><td colspan="2">16</td></tr></table></td>
<td>Practical and visual representations can be used initially to solve missing number problems with division, so children can visualise the relationships when working in the abstract.
E.g. ___÷ 4 =0.5.
What is the whole amount and what are the parts?/How many are in each part?

Look for relationships between numbers to make calculations easier to solve. . E.g. 70÷ ___=3.5.
70÷ 2=35. 35 ÷ 10 = 3.5.
<table><tr><td colspan="2">70</td></tr><tr><td>35</td><td>35</td></tr><tr><td>3.5x10</td><td>3.5x10</td></tr></table>The bar model can show 35x2=70, so how many 3.5 will be in each 35? Use place value counters strips initially and check with a place value slider/grid.
What do you notice?
50÷10=
60÷12=
70÷14=
Why does this happen? Draw bar models to show what is happening.
an you predict other numbers that would follow this pattern?</td>
</tr>
</table>

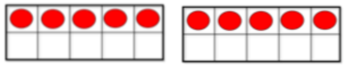
Year 1

Curriculum Links

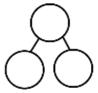
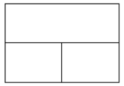
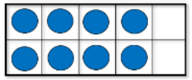
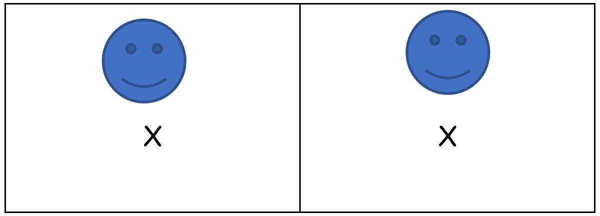
Underlying Skill	Y1 Learning Objectives
Doubling Halving	• Add and subtract one digit and two-digit numbers to 20, including zero. • Solve one-step problems that involve addition and subtraction using concrete objects and pictorial representation and missing number problems. • Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher. • Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs. • Compare, describe and solve practical problems for lengths and heights (e.g. long/short, longer/shorter, tall/short, double/half). • Recognise, find and name a half as one of two equal parts of an object, shape or quantity. • Compare, describe and solve practical problems for lengths and heights, capacity and volume. (e.g. double/half). • Tell the time to half past the hour • Describe position, direction and movement, including whole, half, quarter and three-quarter turns.
Bonds to 10 Bridging Subtraction from a Multiple of Ten	• Add and subtract one digit and two-digit numbers to 20, including zero. • Represent and use number bonds within 20 and related subtraction facts. • Solve one-step problems that involve addition and subtraction using concrete objects and pictorial representation and missing number problems. • Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.
Repeated Addition/ Skip Counting	• Count in multiples of two, five and ten. • Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
Scaling	Scaling is an important context for multiplication and must start early if children are not to become over-reliant on skip counting. Therefore, contexts for doubling should be included in tables and problem-solving work.

Year 1 Guidance

Underlying Skill: Doubling

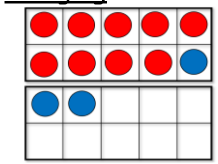
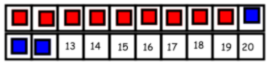
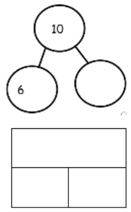
Notes	Concrete	Visual to support Abstract
Ensure doubles are covered before multiplication. Expectation that children know doubles to 10+10. Lots of singing/ doubles raps/ fingers (for doubles within 10). Vocabulary- 'lots of' etc to be used with doubles. Bridging can also be used to develop mental doubling. (See page 18).	Use a range of concrete apparatus, e.g. Numicon, small world objects to show doubles. Show me double 3 with Numicon, with counters, with teddies etc. Use fingers. Show me 4. Now show me double 4. If 5+5=10, what would 6+6 be? How do you know?  Show doubles in an array. Make one half of the array. Ask children to predict the whole amount and add the other half to check. Include and discuss examples of two numbers that aren't doubles (unequal groups). E.g. Which set shows double 5? How do you know?  Make an even number but hide half under a pot/cover. Show half. Predict how many will there be altogether when remove pot/cover?  Hide half of a snake made of beads or a tower of cubes behind your back. This is half my snake/tower. What does my whole snake/tower look like? (Can play as game in pairs). Dice can be used to double. E.g. Can children quickly say what the double is when shown one die? Can they recognise the doubles dominoes and match them to the total, calculations? etc.	Use dominoes to show a double and ask children to complete picture and answer.  Double 1 = _____ Number sentences could be match to dominoes or dominoes drawn to match number sentences. Draw the domino for double 5. How many spots do you think will be on double 6? Explain your thinking. Show one half of an array.  Ask children to imagine then draw the other half and predict what the double will be. Write the addition to match.

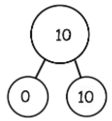
Underlying Skill: Halving (Numbers)

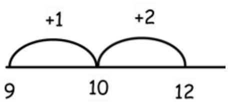
Notes	Concrete	Visual to support Abstract
This section focuses on those aspects of halving which directly underpin relationships in multiplication. However, halving of objects and shapes must also be covered in Year One. Halving should be explored as sharing equally, but also as breaking/folding in two so that two equal parts result. Ensure halves and doubles are closely linked through practical and visual representations.	Use a range of concrete apparatus to halve intially by sharing in real life contexts and on part whole models and bar models. What is the whole amount?How many parts? What is important about the parts?   Find half of 8. Do you need to share here?  Give cubes in two different colours. Build me a tower that is half white. How can you check it is half white? Practise building and snapping in two to make two equal parts. (Link to taller or longer/shorter). Double sided counters could also be used. Make half red.  This is my tower. What would half my tower look like? Predict how tall your tower will be when snapped in half. Build a tower and hide half behind your back. Show the other half and ask children to tell you how many cubes are in the whole tower. True or false. I can break any tower I make exactly in half? Ask children to prove what they say and begin to predict which towers can't be halved exactly and which will make two equal parts. Can they explain what they found?	Relate problems to everyday life, sharing sweets etc. Begin to record by drawing.  When recording visually, it can be helpful to share in an array so that the equality of the parts when halving is obvious. Children can begin to match a range of number sentences to concrete apparatus and visuals such as part whole models or bar models. E.g. $\frac{1}{2}$ of 10= Half of 10 = 10 shared between 2 = Children can also make up and solve their own number sentences to match problems and visuals or draw what they did with apparatus to solve problems. E.g. I make a tower. When I halve it, I have 4. How tall is my tower?

Underlying Skill: Bonds to 10 (Preparation for Bridging)

<table>
<tr><th>Notes</th><th>Concrete</th><th>Visual to Support Abstract</th></tr>
<tr>
<td>Reinforce bonds to 10 throughout the year. Predict the pattern when writing numbers that bond to ten as this involves mentally moving the numbers, so is useful to prepare for bridging.
When children are secure with number bonds they can move onto bridging with practical equipment, again predicting what will happen when the numbers are moved over to make ten.

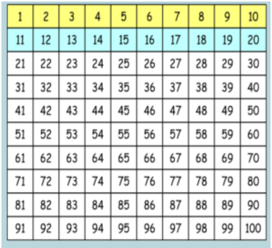
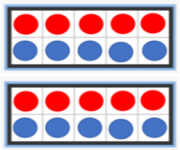
Bonds can also be constantly practised through songs, rhymes, stories and actions.</td>
<td>Place a range of real objects on part whole and bar models, discussing which is the whole amount and which are the parts.
Put 10 counters in the ‘whole’ of the part whole model and ask children to close their eyes while you split them into the two parts. Hide one part under a tub and ask the children to say how many are hiding. How do they know?
Pour out ten double sided counters and find the number sentence to match it, explaining why they chose it. Discuss the different ways children may ‘see’ it, e.g. 3(red) +7(blue), 3(blue) +7(red), 7(red)+3(blue).
Place all the counters the same way up and turn one over at a time. Record systematically, asking the children to predict the next in the sequence. Eg. Turn one over, what will the number sentence be now? (1+9=10). Turn another over, record the number sentence (2+8=10). What do they think the number sentence will be if they turn another counter over? And another? This visualisation requires children to visualise the movement of numbers, so prepares for bridging.
<u>Bridging</u>

Use tens frames alongside number tracks/ 100 squares with cubes placed on top to show how to use the ten as a ‘bridge’ to rest at instead of counting on in ones. E.g. For 9+3, one of the 3 can be imagined moving onto the tens frame so the calculation becomes 10+2 instead of 9+3. It can then be physically moved. Children can predict what will happen then find out.
Begin by adding a single digit to 9, e.g. 9+3, 9+4,9+5 etc. Gradually add to other single digit numbers once confident.

</td>
<td>

Fill in missing numbers on part whole models/bar models.
Use them to work out the parts and the whole in problems.
__ + 6 = 10
6 + __ = 10
10 = 6 + __
Predict and write the next number sentence each time. E.g. What will happen if we move one across (from the right part to the part on the left)?


0+10=10
1+9=10
2+8=10
Explain why one side increases by one each time and the other decreases by one (because one is moving over from one side to the other). Use cubes on bar models or part whole models to show this happening.
<u>Bridging</u>
Teacher can also demonstrate on a numbered


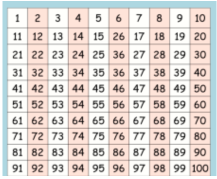
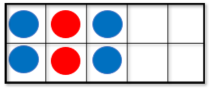
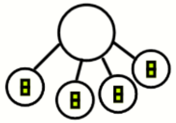
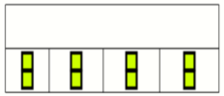
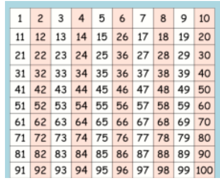
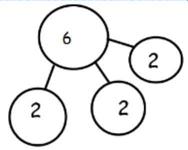
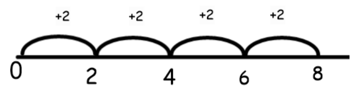
number line.</td>
</tr>
</table>

Underlying Skill: Subtraction from 10/Multiple of 10.

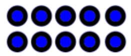
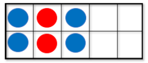
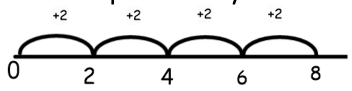
Notes	Concrete	Visual to Support Abstract
Practise counting across the hundred square.	Show in a range of ways: full tens frames, sticks of ten, cubes horizontally across a 100 square etc.   10-2=8. What would 20-2 be? I know 10-4=6. What else do I know? What would 20-6 be? Prove it. What about 30-6?	Write the multiples of ten on an empty 100 square. Use 10-2 to help find 20-2, 30-2 etc and write them in. 10-2=8, 20-2=18 30-2=28. What do you notice? What would be next in the pattern? Use flashcards of full tens frames. Use the first to find 10-6. Predict what the number sentence will be when another ten is added and another etc.

- Guidance on inverse operations can be found on pages 11-14.

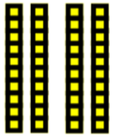
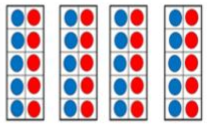
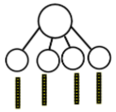
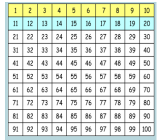
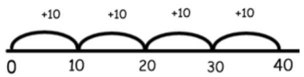
Counting in Twos- Relationship Between Repeated Addition and Skip Counting.

Notes	Concrete	Visual to Support Abstract
The Year One curriculum does not state that children must be be able to use the multiplication sign, but instead focuses on the concept of repeated addition. To prepare for multiplication, describe repeated addition calculations in terms of how many 'lots of' or how many equal groups, before introducing the x sign. Songs are also useful.	Give real life problems involving addition of two each time and use apparatus to represent and solve in lots of different ways, e.g. socks, money etc.    Place counters/cubes/ Numicon twos over top of a 100 square. Write or find the repeated addition to match. How many 'lots of' 2? Combine equipment in part whole models and bar models. Explore which are the parts and which is the whole. Predict the whole amount. Give the whole amount and see how many equal groups of two can be made.   True or false – is 2 + 2 + 2 = 3 lots of 2? Explain. Place twos along the counting stick or number track/number line and mark on the multiples of two. Hide one number for the children to 'guess', explaining how they know. Remove multiples and use to practise counting forwards/backwards from different starting points and to find missing numbers. When children are confident with the concept of repeated addition and describing 'lots of', the multiplication sign can be introduced as a quicker way to express this idea.	Colour to show the pattern. What do you notice?  How many twos are in 20? Circle lots of twos to find out. Show the repeated addition. Circle twos to find a total. Discuss relationships - Which is more – 3 lots of 2 or 5 lots of 2? How do you know? Represent repeated addition visually alongside practical representations. Number sentences could be matched to visuals and visuals to other visuals showing the same calculation. When children are confident with one way of visually recording, begin to encourage them to record practical work in different ways. In an array XXXX XXXX On a bar model 8 2 | 2 | 2 | 2  

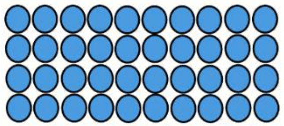
2x Table: Investigating Relationships (Scaling – Doubles Link).

Notes	Concrete	Visual to Support Abstract
Children should begin to recognise that counting in twos can mean lots of 2 or double the amount/twice as many.	Solve real life problems involving doubles using a range of concrete apparatus. Use arrays to reinforce the link between counting in twos and doubling.  When introducing arrays, make one lot of 2, then two lots of two/2+2 etc so children see the pattern that emerges. Ask them to make the next line in an array and describe it (e.g. or 2+2+2 or three lots of two). Can they predict what the next number sentence will be? Eg. Laura thinks this shows 2 + 2 + 2 + 2 + 2 but Ravi says it shows 5+5. Who is right? Show me with cubes, a tens frame, counters etc. Children can make their own arrays using counters to show 2+2+2 and double 3. What do they notice? Tens frames are also useful to demonstrate the link and for children to begin drawing their own arrays. 	True or false: 2 lots of 5 = 5 lots of 2? Draw an array on a tens frame to prove it. Group spots or crosses within an array to show double 5. Now draw around a duplicate of the array to show 5 lots of 2. Write a number sentence to go with each. Does this number line show 5 lots of 2 or 2 lots of 5? Explain how you know.  Could you draw a number line to represent 2 lots of 5? Match number sentences to visuals or concrete representations.

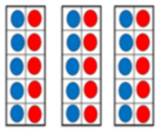
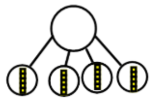
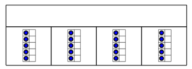
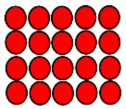
Counting in Tens- Relationship Between Repeated Addition and Skip Counting.

Notes	Concrete	Visual to Support Abstract
Activities are broadly similar to counting in twos. The emphasis is on children becoming confident with the concept of repeated addition and ten times facts, so use the term 'lots of' as well as the x sign, to encourage understanding.	Give real life problems involving repeated addition of ten and use concrete materials to represent and solve. Match apparatus to repeated additions and vice versa. How else could we say it? 10+10+10+10 or 4 lots of 10.     Place tens horizontally across 100 square to show relationship. On empty 100 square, what would 4 tens come to? How many tens are in 40? Prove it.  Put tens along a counting stick or number line and mark on the multiples of ten. Discuss which numbers would be in between them. Jump in tens along a numbered number line and count up by skip counting on a counting stick. (See page 7). You may want to use a bead bar to begin with, so children can physically see the ones within the tens. True or false: 10 + 10 + 10 = 3 lots of 10 ? Make something to show me.	 Colour to show the multiples of ten or to solve problems. E.g. colour to find 10+10+10. Write a repeated addition or find a different visual which shows the same calculation. Give out cards with multiples of ten on them. Muddle them up and ask children to reorder and place on an empty number track or line. Which numbers would go in between them? Draw number lines to solve repeated addition or 'lots of' facts. E.g. Draw 4 lots of ten on the number line. 

10x Table- Investigating Relationships

Notes	Concrete	Visual
Activities broadly similar to that of 2x table, as emphasis is on children becoming confident with multiplication facts.	Make arrays (see page 21) to show different multiplication/repeated addition calculations. Use to discuss the commutative property. E.g. Discuss how this can show both 10x4 and 4x10 (10+10+10+10 or 4+4+4+4+4+4+4+4+4+4). Match repeated addition and 'lots of' and multiplication calculations to concrete representations and to each other. 	Draw arrays to match repeated additions and vice versa. Circle counters/crosses in arrays to show the number sentence given.

5x Table- Relationship Between Repeated Addition and Skip Counting

Notes	Concrete	Visual to Support Abstract
Activities broadly similar to counting in twos, as emphasis is on children becoming confident with the concept of repeated addition. See page 21 for introducing arrays.	Use a range of practical apparatus to 'act out' and represent problems.  Write the repeated addition to match a representation. Use 10s frames to help make links between 5s and 10s. Place cubes in fives on top of a hundred square to show the pattern.   Put practical equipment which represents fives (e.g. 5p coins/cubes etc) in part whole models and bar models. Discuss which are the parts and which is the whole. Use to solve repeated addition calculations and also describe in terms of 'lots of.' Make arrays to show different repeated addition calculations. Use to discuss the commutative property.  E.g. Discuss how it can show both 5 lots of 4 and 4 lots of 5, 5+5+5+5 or 4+4+4+4+4. Match calculations to concrete representations and vice versa. True or false – Is 5 lots of 3 = 3 lots of 5? Explain how you know. Place concrete apparatus along a counting stick and discuss the relationship between repeated addition and skip counting.  Jump in fives along a counting stick and count up by skip counting. (See 'The Counting stick', page 7). Make two sticks of ten and break them in half. How may fives have you got? What do you notice? What do you think will happen with three tens?	Colour in the 100 square to show patterns and make the link between fives and tens visually.  Jump in fives along a numbered number line and count up by skip counting. Match different visual representations (array, bar model, part whole model, number line) to repeated addition and multiplication calculations or calculations to different visuals. True or false: When I count in fives, the numbers I say will always have a five in the ones? Sort cards with 2 digit numbers on them according to whether or not they are a multiple of five, checking with concrete apparatus. Explain what you notice. Predict other numbers that you would say when counting in fives. Put cards on 100 square to check.

Year 2

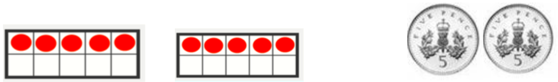
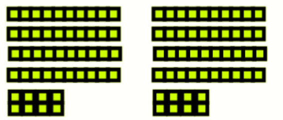
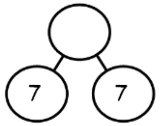
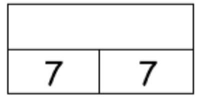
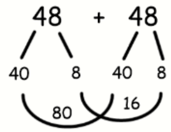
Curriculum Links

Underlying Skill	Linked to Y2 Learning Objectives
Doubling and halving	• Recognise the place value of each digit in a two-digit number (tens, ones) • Identify, represent and estimate numbers using different representations, including the number line • Use concrete objects, pictorial representations, and mental methods to add and subtract a two-digit number and ones, two-digit number and tens, 2 two digit numbers and 3 single digit numbers. • Recall and use addition and subtraction facts to 20 fluently and derive and use related facts up to 100 • Recall and use multiplication and division facts for the 2 multiplication table • Recognise odd and even numbers
Subtraction from a Multiple of Ten (Teach this before teaching bridging)	• Identify, represent and estimate numbers using different representations, including the number line • Use place value and number facts to solve problems • Recall and use addition and subtraction facts to 20 fluently and derive and use related facts up to 100 • Recognise the place value of each digit in a two-digit number (tens, ones)
Bridging	• Identify, represent and estimate numbers using different representations, including the number line • Use place value and number facts to solve problems • Use concrete objects, pictorial representations, and mental methods to add and subtract a two-digit number and ones. • Recall and use addition and subtraction facts to 20 fluently and derive and use related facts up to 100. • Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs. • Show that multiplication of two numbers can be done in any order and (commutative) and division of one number by another cannot • Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
Repeated Addition and Skip Counting	• Count in steps of 2, 3 and 5 from 0, and in tens from any number • Recall and use multiplication and division facts for the 2, 5 and 10 multiplication table • Identify, represent and estimate numbers using different representations, including the number line • Use concrete objects, pictorial representations, and mental methods to add three one-digit numbers

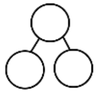
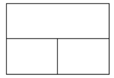
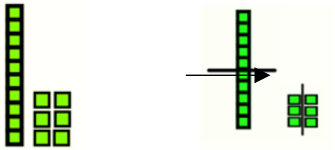
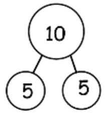
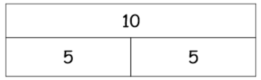
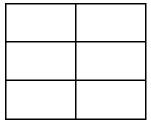
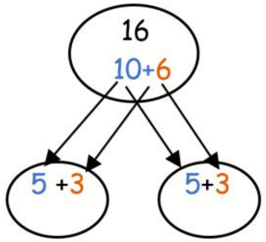
	• Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs. • Show that multiplication of two numbers can be done in any order and (commutative) and division of one number by another cannot • Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
Investigating Properties and Relationships	• As above. • Use concrete objects, pictorial representations, and mental methods to add three one-digit numbers

Year 2 Guidance

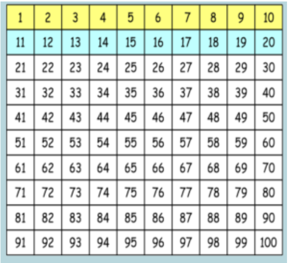
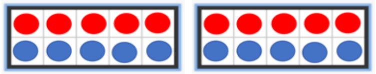
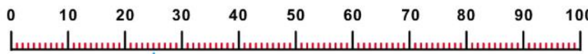
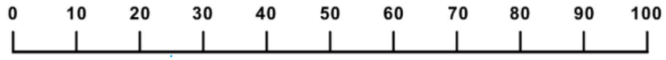
Underlying Skill: Doubling

Notes	Concrete	Visual to Support Abstract
Revise doubles up to 5+5 and use with partitioning/place value to work out new facts up to 10+10. Extend and consolidate strategies throughout work with 2 digit numbers. Doubles may also be found by bridging (see page 29). As part of the work on doubles, teach children to visualise and exploit number relationships, by making use of near doubles: Show 6+6. What would 6+7 be?	If 5+5=10, what would 6+6 be? How do you know?  What about 7+7? (Draw attention to it as 5+5+2+2, so known facts can be used). Represent with apparatus in part-whole models and bar models. Use a range of apparatus. Hide half for children to guess the whole amount. E.g. This is half my tower. What does my whole tower look like? Prove it.  Build in opportunities to consolidate doubling throughout addition work. E.g. Addition of 3 single digits → 6+3+6 2 digit+1 digit → 36+6 2 digit +2 digit → 48+48 =80+16 	 Show a picture of half of an amount. Draw the rest of the objects/tower/number line etc. Put into bar models and part whole models. Show on number line.  What is double 48? How did you work it out?  Did anyone find it a different way? (E.g. 48+40=88. 88+8).
Reasoning (to address misconceptions/mistakes or make predictions and generalisations).	• Erin says 46+46=86. What is her mistake? • True or false: double 37>60. How do you know? • Double 7<_____. What could it be? What couldn't it be? • Tianna says double 27 is 56 but Rory says he knows this can't be correct. Explain how he knows. (Doulbe 6=12 so there should be 2 in the ones).	

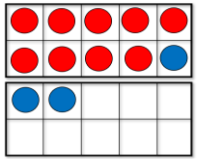
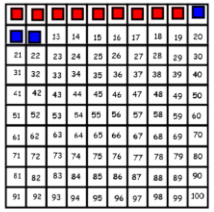
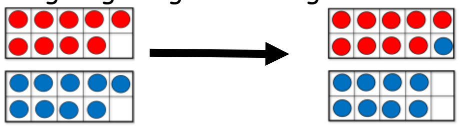
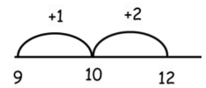
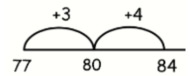
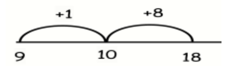
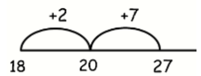
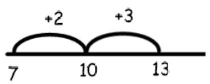
Underlying Skill: Halving

Notes	Concrete	Visual to Support Abstract
Use equipment where the ten group is obvious, such as tens frame, Numicon or cubes in sticks of ten with ones. Halving will also be carried out with shapes and other objects, but for the purpose of times tables, this programme focuses on number only. It is important to move beyond sharing out apparatus and use the structure of numbers to build mental strategies.	Use a range of concrete apparatus to halve intially by sharing on laminated part-whole and bar models. What is the whole amount? How many parts? What is important about the parts?   Give cubes in two different colours. Build a tower. What would half your tower look like? Practise building, snapping in two to make two equal parts and comparing each part. Recap halves of numbers up to 10. Build me a tower that is half white etc. How can you check that half is white?  Beau says these tower are half yellow? Is he right? Repeat the above activities with numbers to 20. Use knowledge of halves to ten and place value to halve teens numbers up to 20. What is half of 10? So what would half of 12 be? (Halve the 10, halve the 2). What about 14? 16?  What about half of 26, 84 etc.	  Colour half of a tower picture or grid. Write the fractions sentence to go with it.  Half of 16 
Reason to address misconceptions or help make generalisations.	• David has 26 cubes. He says if he gives half away, he will have 16 left. Explain his mistake. Prove it. • Jess has three towers of bricks. One is 14 cubes tall, one is 15 cubes tall and one is 17 cubes tall. Half of one tower is white. Which tower? Explain how you know.	

Underlying Skill: Subtraction from a Multiple of Ten

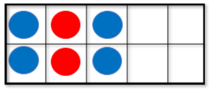
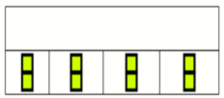
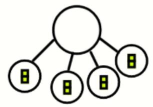
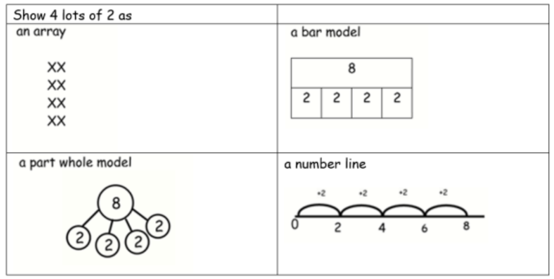
Notes	Concrete	Visual to Support Abstract
This skill links to the later strategy for finding the ninth multiple, so it must become automatic. It also consolidates number bonds to ten.	Show this in a range of ways: full tens frames, number pieces, sticks of ten cubes horizontally across 100 square etc. E.g. 10-2=8. What would 20-2=   Give an empty 100 square. Write on the multiples of 10. If 10-2=8, write in 20-2,30-2 etc.  Look at the number line. If I jump back 2 from 10, where will I land? What about 20,30 etc?	Write the multiples of ten on an empty 100 square. Use it work out 10-2, 20-2, 30-2 etc. 40-6= 10 10 10 ~~10~~ -6= 4 Draw place value counters to work out, gradually moving towards mental methods. Show where 30-2, 40-2,50-2 etc will be on an empty number line. 
Reason to address misconceptions or generalise.	I use my empty 100 square/number line to work out that 50-4=56. What mistake have I made? Use concrete equipment to prove it. Look at the pattern: 10-3=7 20-3=17 30-3=27 What would come next? Prove it. I know 10-6=4, what other subtraction facts can I work out? Can these help me find any addition facts?	

Underlying Skill: Bridging

Notes	Concrete	Visual To Support Abstract
This reinforces the key skills of number bonds, partitioning and place value, whilst allowing children to manipulate numbers and develop mental methods. Before learning to bridge with 2 digit numbers, children must be able to identify the ten before and after it. They must also be able to add single digits to multiples of ten and know how many would be needed to reach the next ten.	Use tens frames alongside number lines and 100 squares with cubes placed on top to show how to use the ten as a 'bridge' to rest at.  Begin by adding a single digit to 9. Show it on 2 tens frames or with 9 counters on a number track to 10 and 3 other loose counters. Ask the children to predict what the total will be when put together, so they can visualise the movement of the counters. Then physically move it, so 9+3 becomes 10+2.  Repeat with 9+4,9+5 etc. Gradually introduce other numbers (8+3,8+4 etc) once confident. It can be also explored through placing different coloured cubes on top of tens frames or 100 squares. The strategy can then be used with larger numbers, e.g. 18+4, 27+5, 79+7 when adding single digits to 2 digit numbers.  Bridging can also be used to develop mental doubling. What would 9+9 be? Explain how you know. Once established, build in opportunities to triple numbers by doubling then bridging the last number, e.g. 8+8+8 as this lays strong foundations for the three times table, as well as consolidating doubles. Bridging can be taught as part of the work on addition of single digits to 2 digits numbers.	Colour squares on a 100 square to show cubes added or draw counters added in a tens frame. An arrow can be used to show how one of the 3 has moved to the 9 to make an equivalent (easier) calculation – 10+2. 1 9 + 3 = 10 + 2 It is also useful to show it on a number line, as children can draw their own as a stage between the concrete and the abstract to help them imagine the process. 9+3  77+7  9+9  9+9+9 
Reason to address misconceptions or to generalise.	David worked out 7+5 like this. Is he right? Explain your thinking.  	

- Guidance on inverse operations can be found on pages 11-14.

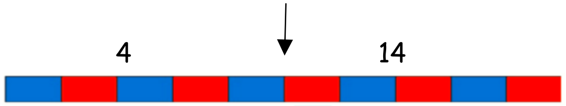
2x Table: Relationship Between Repeated Addition and Skip Counting

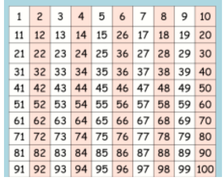
Notes	Concrete	Visual to Support Abstract
Make the point that reading and recording repeated addition calculations is time consuming, so it can be referred to 'lots of', The multiplication sign can be shown to represent the 'lots of'. It is important to investigate the relationships between multiples when skip counting.	Give real life problems involving repeated addition of two and use concrete materials to act out the problem and solve. Draw to show.  2+2+2= Match apparatus to repeated addition statements and then multiplication statements and vice versa. Place concrete apparatus in twos along a counting stick and mark on the multiples. Point to a multiple and ask the children to write the repeated addition then the multiplication.   Get out coins to represent repeated addition and multiplication calculations. How much money is there? Write the repeated addition calculation and the multiplication statement. How much would there be if I added 2p more? Spent 2p? etc. Find out. Place cubes over numbers on 100 square to match calculations then use them to solve calculations.  Use part whole and bar models and discuss which are the parts and which is the whole.  Put twos in the parts. Describe the parts and the whole in a number sentence. Now show the whole amount. How many groups of two will you get? Match and later write repeated addition, multiplication and division calculations to match.	Once children are secure with one way to visually represent, explore different ways. E.g. show 3 lots of 2 as a repeated addition, practically and on a 100 square, number track or number line, in an array.  Draw pictures and use visuals to represent and help expose the relationships within multiplication problems and the link to division. Match different representations to calculations and find visual representations which show a given calculation. Make up problems to represent different visuals or calculations. Draw visual representations to solve problems. These may begin as simple pictures then move onto number lines, part whole models etc. Ensure examples include multiplying by 1 and 0.

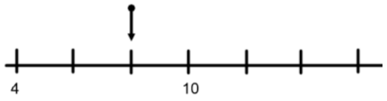
<table>
<tr>
<td></td>
<td>After lots of practical experiences, show how the x sign can be used as a quicker way to express 'lots of'. (If children are confusing the multiplication and the addition sign, use x to cross out 'lots of') as a prompt for those confusing the orientation.

Use the counting stick methods (see page 7) to explore relationships between multiples of 2 and find more efficient ways to derive them using underlying skills and known facts. What do you think 11x2 would be? 12x2? How do you know?

Once confident with the counting stick, the teacher can model relationships and missing numbers on a number line.

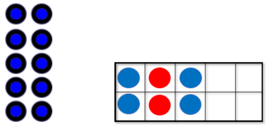
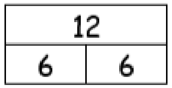
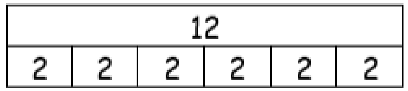
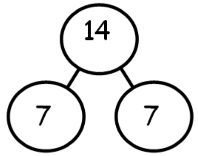
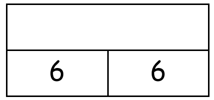
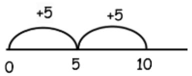
The counting stick can also be used for the reading scales link:


Give out number cards from 1 to 20. Use cubes to put numbers into twos. What do you notice?</td>
<td>Colour to show the pattern of the two times table. What are the numbers you haven't coloured called? What are the multiples of two called? What do you notice about each?

How many twos did you count to reach 10?
How many do you think you will count to reach 20?

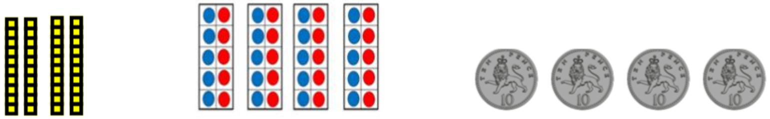
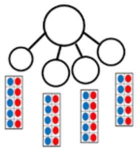
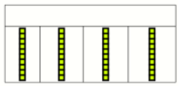
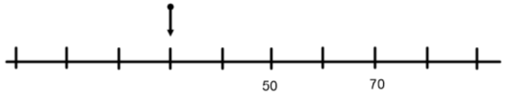
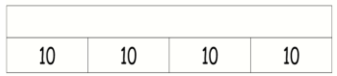
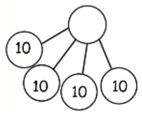
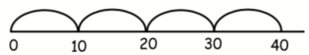
Where would 14 go? What about 13?


Sort the numbers cards which could and couldn't be put into twos. Predict which numbers in the 20s could be put into twos etc.</td>
</tr>
<tr>
<td>Reason to address misconceptions or help make generalisations.</td>
<td colspan="2">
Count in twos to solve this: 2+2+2+2=
What can help you solve this: 2+5+2+2=
True or false: 9x2<4x2? Explain how you know.
Look at these numbers: 10, 20, 30, 40. Which can be put into twos and which can't? Use concrete apparatus to find out.
Which is the odd one out: 23,18,17? Explain why.
</td>
</tr>
</table>

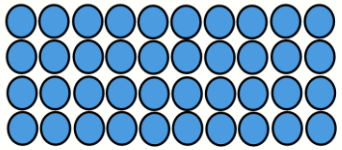
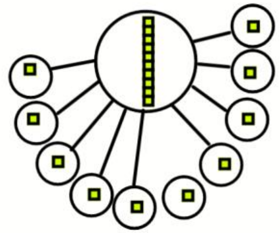
2x Table: Investigating Relationships (Scaling - Doubles Link).

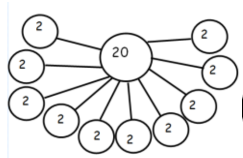
Notes	Concrete	Visual to Support Abstract
Link x2 to doubling, as doubling is needed when multiplying larger numbers by 2 (e.g. 34x2). Teach doubles before x2, so the link can be made. Ensure that some problems involve scaling up by 2 (doubling) as well as counting in twos, using language such as twice as much, double and two times (E. 3x2 can mean 3 two times or 3 lots of 2).	Use apparatus to represent and solve doubles problems.  Investigate relationships between skip counting in twos and doubling by making arrays (see page 21). Match calculations to concrete representations and use apparatus to show calculations in different ways. E.g. show it with coins, cubes etc. True or false- 2 lots of 5=5 lots of 2. Why? Prove it. What would 2x6 look like on a part whole model/bar model? What about 6x2? Use concrete apparatus to make an array to show a given calculation. Write 2 repeated addition and 2 multiplication calculations (and later division) for an array made by your partner. Identify the repeated addition to match a problem. e.g. Does a problem require 4+4 or 2+2+2+2? Given concrete apparatus, make up a problem that it could represent.	Represent problems and calculations visually by drawing circles or crosses in an array, bar models and part whole models. E.g. show 2+2+2+2+2+2, now show double 6. Explain what you drew each time. Write multiplication calculations to match. What's the same? What's different?   Record and solve problems and calculations visually in a range of ways. Write a repeated addition, multiplication, division calculation to match each visual.   Write problems to match calculations and visuals. Explore associated language, making clear that x2 can mean lots of or two times as many.
Reasoning to address misconceptions or help make generalisations.	• Show 7 lots of 2 on a number line/bar model/part whole model. Now show 2 lots of 7. What's the same? What's different? • Does the number line show 5 lots of 2 or 2 lots of 5? Explain.  Always, sometimes, never: double 7 = 7x2? Double 43=43x2?	

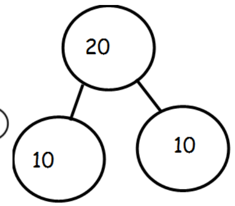
10x Table: Relationship Between Repeated Addition and Skip Counting

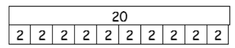
Notes	Concrete	Visual to Support Abstract
Make the point that recording and reading repeated addition calculations is time consuming, so we can refer to them as lots of 10. Count past 100 and show what happens when we reach 10 lots of ten. Ensure examples which include multiplying by 1 and 0 are included.	Give real life problems involving repeated addition of ten and use apparatus to act out the problem and solve.  Write it as a repeated addition. How else could we say it? 10+10+10+10 or 4 lots of 10.  Place tens horizontally across a 100 square to show the relationship. On an empty 100 square, what number would 4 tens come to? How many tens are in 40? Show on a 10x10 peg board. Put out cards with multiples of ten on them. Muddle them up for children to reorder and place on an empty number track or line. Which numbers would go in between the multiples of ten? Check with apparatus. Put apparatus into part-whole models and bar model and discuss which are the parts and which is the whole. Describe them using repeated addition and multiplication calculations. What would the division calculation be?   Use counting stick methods (see page 7) to explore the relationships between multiples of 10 and find more efficient ways to derive them using underlying skills and known facts. What is 11x,12x? How do you know?	 Colour to show calculations, e.g. 2x10. Write the repeated addition and multiplication calculations to match. What number is this pointing to? How could you find out?  Record one calculation in a range of ways – an array, part whole model, bar model, number line etc.     What does the part whole model show? Find a different visual that shows the same calculation. Match calculations to visuals and vice versa. Use different representations to show or solve a problem. Write a problem to show a calculation.

10x Table: Investigating Relationships

<table>
<tr><th>Notes</th><th>Concrete</th><th>Visual to Support Abstract</th></tr>
<tr>
<td>It is important to investigate, not just 10 lots of something, but also idea of scaling by making 10x greater as this is fundamental to the later understanding of place value.
Use language related to scaling, e.g. 10 times as much/many, 10 times greater and give practical problems including these.</td>
<td>Make arrays (see page 21) to show different multiplication/repeated addition calculations. Use them to discuss the commutative property. E.g. Discuss how this can show both 10x4 and 4x10 (10+10+10+10 or 4+4+4+4+4+4+4+4+4+4).
Match calculations to concrete representations.

Extension: Discuss the relationships that exist whilst working with practical equipment. What else could this represent? E.g. 2 lots of 2x10 or (2x10 and 2x10) or 4 lots of 2x5 or 1 lot of 10 and 3 lots of 10 etc.

Investigate what happens when we make ones 10x bigger. Also, show on bar models.
Write the repeated addition. Write the multiplication. What would the whole be if we put 2 in each part? How about 3? etc.
What division calculation would this show?</td>
<td>Match pictures of arrays to calculations and vice versa.
Draw arrays, part whole models and bar models to match calculations and to solve calculations and problems.

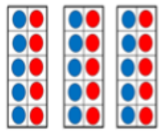
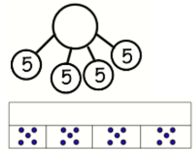
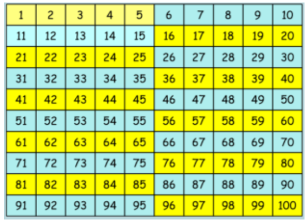
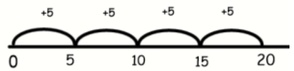
Look at the visuals. What's the same? What's different?




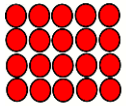
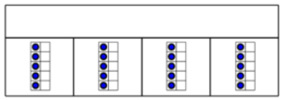
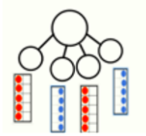
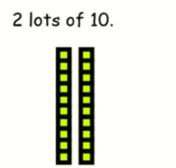
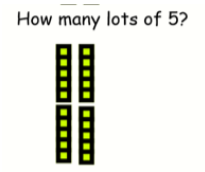
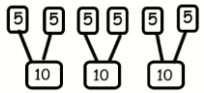
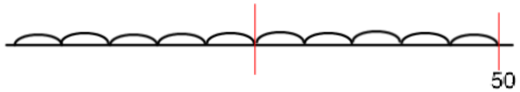
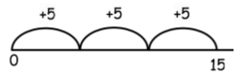


<table><tr><td colspan="2">20</td></tr><tr><td>10</td><td>10</td></tr></table>Write a word problem to go with a visual or calculation.</td>
</tr>
<tr>
<td>Reasoning to address misconceptions or help make generalisations.</td>
<td colspan="2">What do you notice about the multiples of ten? Which is the odd one out? Why?
10, 18, 20, 40, 110.
10x4>_____What could it be? What couldn't it be?
True or false: 10x3<3x10. Explain how you know.</td>
</tr>
</table>

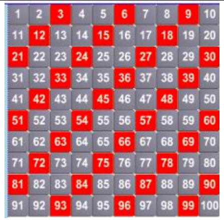
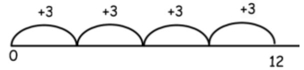
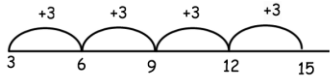
5x Table: Relationship Between Repeated Addition and Skip Counting

Notes	Concrete	Visual to Support Abstract
Make the point that recording and reading repeated addition calculations is time consuming, so we can refer to them as 'lots of' 10. Ensure examples which include multiplying by 1 and 0 are included.	Give real life problems involving repeated addition of five and use apparatus to act out the problem and solve. Draw to show understanding.  Write a repeated addition to show this. How else could we say it? 5+5+5+5+5+5= or 6x5. Can you show this with concrete equipment, e.g. coins? Place cubes in fives over a 100 square. What do you notice? Place cubes along a counting stick and count in fives.   Model and write repeated addition and multiplication. Show how these can be represented as jumps on a number line. Use counting stick methods (see page 7) to explore the relationships between multiples of 5 and to find more efficient ways to derive them using underlying skills and known facts. What do you think 11x5 would be? 12x5? How do you know? Make sure the relationship of x9 to tenth multiple is focused upon particularly and discuss whether this strategy would work for other multiples. E.g. How could you find 9x3? 9x4? The counting stick can also be linked to reading scales in fives and finding missing numbers. What is the missing number? How did you find it? Did anyone find it a different way?  	Represent repeated addition, multiplication and division calculations in a variety of ways (see examples for x2 on page 30). Match calculations to arrays, bar models, part-whole models and number lines and vice versa.  Write the repeated addition and multiplication that these show. Write the 2 division calculations that they show. Use to find missing numbers, e.g. __x5=20. Write a problem that the visual shows or to match a calculation.  Colour a 100 square to show the pattern of fives. What do you notice? Record and solve calculations and problems on an empty number line. 

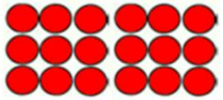
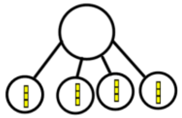
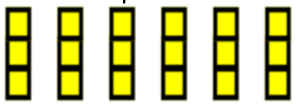
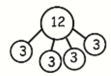
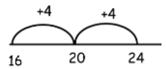
5x Table: Investigating Relationships.

Notes	Concrete	Visual to Support Abstract
It is important to exploit doubling and halving knowledge to investigate the relationship between the fives and the tens. Use language related to scaling, e.g. 5 times as much/many, 5 times greater and give practical problems including these. When looking at multiplication, ask what it could mean. E.g. 5x6 could be: 5 lots of 6, 6 lots of 5, 5 six times, 6 five times, 6 multiples of 5, 5 multiples of 6,5+5+5+5+5+5, 6+6+6+6+6 etc.	Make arrays (see page 21) to show different multiplication/repeated addition and division calculations. Use these to illustrate the commutative property. E.g. Discuss how it can show both 5x4 and 4x5, 5+5+5+5 or 4+4+4+4+4. Match calculations to concrete representations.  Extension: What else could it show? (2x10, 3 fives and another five). Show me two fives and another two fives. How many fives have you got?   Investigate the commutative property. Make a bar model to show 4x5. Now make one to show 5x4. What's the same and what's different? Make up a problem for each. Investigate the relationships between tens and fives practically and combine in part-whole models and bar models. Play as a game. Show 3 lots of 10. Predict how many lots of 5? How many will there be in 4 tens? Prove it using cubes, tens frames and other number equipment. Show it on a 100 square.   Explore the relationship between 10x and 5x on a counting stick, putting concrete apparatus along the stick.  If 10x5=50, what is 5x5? What if it were 10x2? 10x10?	Draw and group arrays to show different multiplication and repeated addition calculations. E.g. 5x4, 4x5, 5+5+5+5 or 4+4+4+4+4. Also, link arrays to division. Make the link between the fives and the tens.  3x10= __x5. 10x5=50, what is 5x5?  What tables fact does the number line show? How can it help you to find 6x5? What do you think 12x 5 would be?  This shows 3x5. Predict 6x5. Draw a number line to show 3x5+3x5. True or false? 6x5>2x5. 4x5> 4x2. Explain how you know. How many different ways can you find the answer to 8x5? (Discuss the different ways to help children understand the relationships within multiplication tables, e.g. count in fives, or 8+8+8+8+8 using doubles and addition, or ten fives subtract two fives or double four fives).

3x Table: Relationship Between Repeated Addition and Skip Counting

Notes	Concrete	Visual to Support Abstract
Although the 3x table is covered in Year 3, counting in threes is covered in Year 2 and there have been questions on Year 2 SATs papers testing the 3x table. Therefore, it makes sense to cover it in Year 2 and revise it in Year 3. Ensure examples which include multiplying by 1 and 0 are included.	Represent and solve problems involving repeated addition of three using concrete apparatus.  Write a repeated addition to show this. How else could we say it? 3+3+3+3 = or 4x3= Can you show this with cubes? Place cubes/number pieces/Cuisenaire rods along a counting stick and mark in threes.  Use counting stick methods (see page 7) to explore the relationships between multiples of 3 and find more efficient ways to derive them using underlying skills and known facts. What do you think 11x3=? 12x3? How can 10x3 help you find 5x3? 9x3? Make sure the relationship between the ninth and tenth multiple is focused upon particularly and discuss whether this strategy would work for other multiples. E.g. Could you use it for 9x4? 9x6? Model and write repeated addition and multiplication calculations. Begin to derive the division facts using counting stick. E.g. How many threes in 15? Use an empty counting stick to link facts together and revise other tables. E.g. Point to the tenth multiple and ask if this is 30, what would 9x be? What about if the tenth multiple was 50? 100? 20?	 Colour a 100 square to show the pattern of threes. Can you spot any patterns? Use counting stick relationships to find missing numbers on empty number lines. Draw number lines to solve calculations and problems. What tables fact does the number line below show? Can you use this to find 8x3? Rory says it also shows 2x3+2x3. Is he right?  Match repeated addition and multiplication calculations to number line visuals and vice versa. Make up a problem that a calculation or visual could show. Gemma drew this number line to show 4x3. What is her mistake? 

3x Table: Investigating Relationships.

Notes	Concrete	Visual to Support Abstract
Use language related to scaling as well as skip counting. E.g. 4x3 can be 4 lots of 3 or 4 three times or triple four. Tripling is important, as it helps to show the commutative property and encourages children to think about tables fact from different perspectives.	Make arrays (see page 21) to show different multiplication/repeated addition calculations. Use them to discuss the commutative property. E.g. Discuss how it can show both 6x3 and 3x6, 6+6+6 or 3+3+3+3+3+3. Match calculations to concrete representations and to visuals. What division facts can you work out?  Extension: What else could it show? (2x9, 9x2). Show me threes threes and another three threes. How many threes have you got? How could we write a calculation to show this?   Make a bar model/part whole model to show 4x3. How many parts will you have? How many in each part? What will the whole be? Now make one to show 3x4. What's the same? What's different? Can you make up a problem for each representation? Can you use the bar model above to help you work out 8x3? 2x3? What would your bar model look like if you wanted to show 4 three times/three times as many? Investigate relationships practically with discussion and prediction of what will happen when numbers are changed, doubled etc. Combine in part-whole models and bar models practically.  Show 6 lots of 3. Predict 12 lots of 3, 3 lots of 3? Prove it using concrete equipment. Show it on a 100 square/counting stick/ number line.	Show 3x4 In an array | In a bar model In a part whole model | On a number line   Use visuals to solve missing number problems and investigate the relationship between multiplication and division. Write multiplication and division sentences represented by visuals. What do you notice: 2x3=6 4x3=12 Which calculation would come next? Explain. Show a calculation. What could it mean? E.g. 3x8 – 8 lots of 3, 3 lots of 8, 8 three times, 3 eight times, 3 multiples of 8 etc. Discuss different ways to solve. Practise tripling, which makes use of doubling and bridging. E.g. e.g. 3x8 - 8+8+8 16 +8 
Reasoning	• True or false: 6x3>2x3. Explain how you know. • 3x10> _____. What could go there? What couldn't go there? • 3x3 _____ 7x3? Which of these could not go in the middle: 5x3, 3x4, 3x2. Explain how you know.	

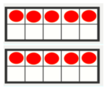
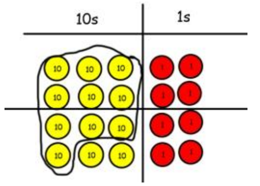
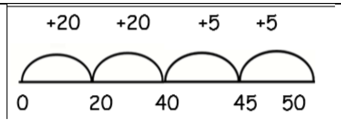
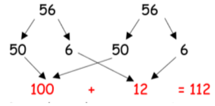
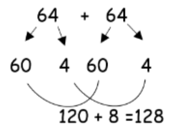
Year 3

Curriculum Links

Skill	Linked to Y3 Learning Objectives
Doubling (near doubles) and halving Multiplication/ Investigating Relationships	• Identify, represent and estimate numbers using different representations. • Add and subtract numbers mentally: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds • Add and subtract numbers with up to three digits, using formal written methods of columnar addition. • Solve problems including missing number problems, using number facts, place value, and more complex addition and subtraction • Count from 0 in multiples of 4, 8; count in multiples of 50 and 100 • Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables • Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods. • Estimate the answer to a calculation and use inverse operations to check answers. • Solve problems, including missing number problems, involving multiplication and division • Solve problems including positive integer scaling problems and correspondence problems in which n objects are connected to m objects. • Recognise and show, using diagrams, equivalent fractions with small denominators. • Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators. • Recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators • Solve problems that involve all of the fraction skills taught • Count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10.
Subtraction from Multiple of 10 and 100 Bridging	• Add and subtract numbers mentally: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds. • Add numbers with up to three digits, using formal written methods of columnar addition. • Subtract numbers with up to three digits, using formal written methods of columnar subtraction.

Year 3 Guidance

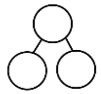
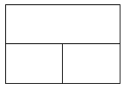
Underlying skills: Doubling/Near Doubling

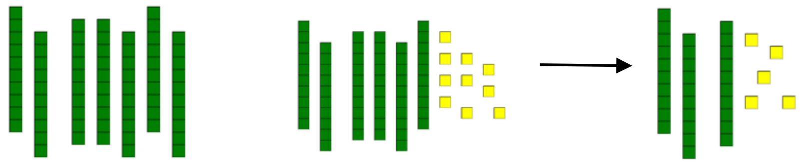
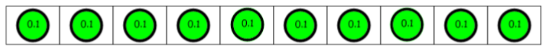
Notes	Concrete	Visual to Support Abstract
Revise doubles up to 12 + 12 using strategies from Year 2 if necessary (p26). Also include near doubles. Double multiples of 10, using what is known about single digits. Double multiples that end in 5, (25, 45 etc) Build in opportunities to double/near double when working with place value, addition, subtraction money and measures. Also, include when using formal written methods, so children continue to use mental skills. When doubling, link to x2, so children develop mental methods for x2.	If 20+20 = 40, then what is 25 + 25? Use Dienes or place value counters to prove it.   What does this show? What if each of the counters was worth 10/was replaced with a tens counter? What would 60+60 be? How do you know?  These represent half my sweets. How many sweets do I have in total? Predict, then prove it. Double 64 = 100+20+8.  What number facts could help me find double 64? (Link double 6 to double 60 using practical equipment to begin with).	 What is double 56? How did you work it out? Did anyone find it a different way?  Place value counters may be drawn alongside to illustrate for a short period, if relationships are not fully understood. If double 56=112, what would 56+57 be?  So, what would 64x2=? Revise doubling throughout all tables by exploring and making decisions about different ways to solve calculations, e.g. 4x9 = 9+9+9+9 
Reason (to address misconceptions or to make predictions/ generalisations).	• True or false: double 87>140. How do you know? • Double 12<_____. What could it be? What couldn't it be? • Sally says 46 +46 = 82 but Rory says he knows this can't be correct. How does he know? • Asif has twenty six 2p coins. He thinks he can find out how much he has by doubling. Is he right?	

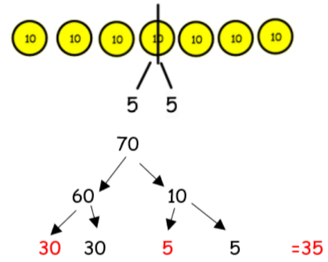
Underlying skills: Halving

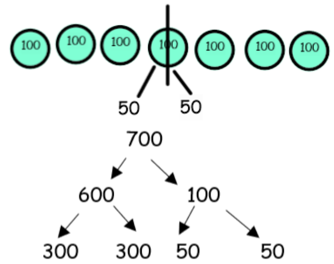
<table>
<tr><th>Notes</th><th>Concrete</th><th>Visual to Support Abstract</th></tr>
<tr>
<td>Use equipment such as Dienes or place value counters to link 2-digit numbers to known facts.

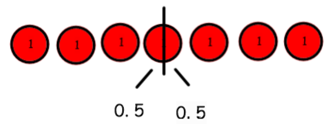
If children are not secure from Year 2, build in opportunities to halve 2-digit numbers by halving the tens and halving the ones.
Ensure two-digit numbers with an odd number of tens are included.

Doubles and halves relationships can be revised when exploring equivalent fractions.</td>
<td>Use a range of concrete apparatus to solve problems and halve - intially by sharing. What is the whole amount? How many parts? What is important about the parts? Write word problems that different representations/ calculations could represent.
 
Share place value counters in an array (see pages11-14) to show the link between dividing by 2 and halving.

E.g. Half of 70. Use Dienes to show exchange in the final 10.

Also use place value counters and predict what will happen to the last ten.
Halve 2 digit numbers where the number of tens is odd, e.g. 36, 54, 78 (e.g. Halve the 30, halve the 6 etc).
Use a counting stick, Cuisenaire rods etc to find out how many halves are in 2? 3? 4? etc. What do you notice? Why does this happen?


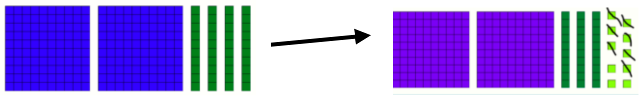
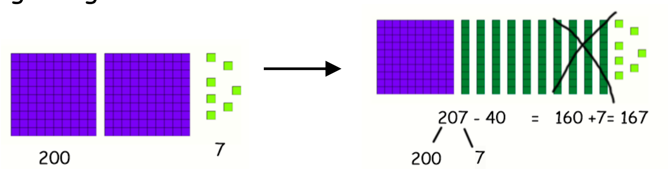
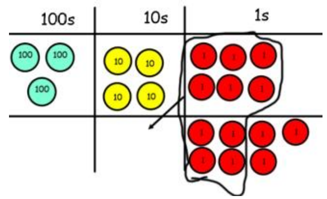
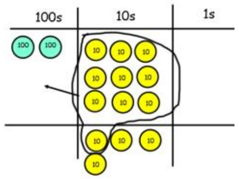
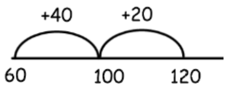
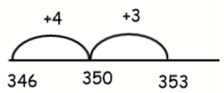
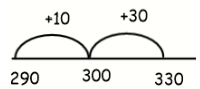
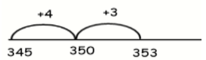
Use decimal place value counters when exploring tenths. Find half of one. How can we write it as a decimal?
Halve odd numbers, showing 0.5, e.g. half of 3 = 1.5 or $1\frac{1}{2}$.</td>
<td>E.g. Half of 70. Draw Dienes or place value counters.


What would half of 700 be?


Half of 7?


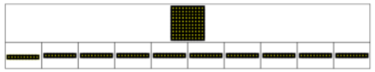
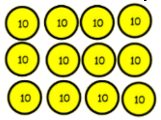
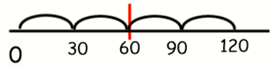
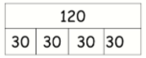
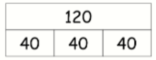
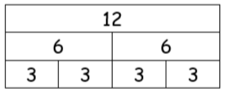
Write down all the fractions equivalent to half. What do you notice?</td>
</tr>
<tr>
<td>Reason to address misconceptions or help make generalisations.</td>
<td colspan="2">• David has 96 cubes. He says if he gives half away, he will have 43 left. Is he right? Explain his mistake. Prove it.
• Jess has three towers of bricks. One is 14 cubes tall, one is 15 cubes tall and one is 17 cubes tall. Half of one tower is 7.5. Which tower? Explain how you know.</td>
</tr>
</table>

Underlying Skills: Subtraction from Multiples of 10 and 100 and Bridging.

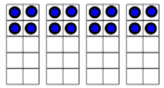
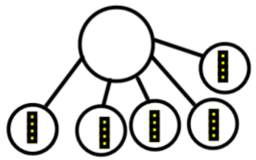
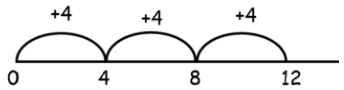
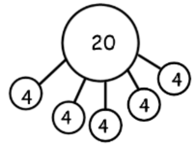
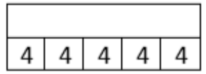
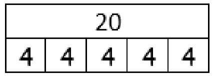
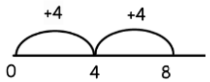
Notes	Concrete	Visual to Support Abstract
Subtraction from multiples of ten is required to be automatic for x9 by subtracting to find the ninth multiple and for recognising relationships when estimating, multiplying and dividing with larger numbers. (E.g. knowing that there will be 24x4= 96 because 25 x4=100). Bridging is required for x3 by tripling.	Discuss strategies during formal written methods, so children learn to decide when to work mentally and when to use a written method. Use Dienes or place value counters to make the link to known facts. 3-digit -1 digit – E.g. 240-7 or 300-4.  E.g. 3 digits –tens – 207-40.  Bridging 3-digit +1 digit - 346+7  3 digit + tens – 290+40. 	Draw the Dienes or place value counters to make link to known facts. E.g.240-7=233 because 10-7=3.  Also, show on number lines. Bridging Links can be made to single digit facts when working with: multiples of ten.  3-digit +1 digit - 346+7  3 digit + tens – 290+40.  Doubles can also be consolidated during this work.
Reasoning - address misconceptions or make generalisations.	Link to known facts, I know 40-7=3 so 240-7=? What would 340-7 be? 300-4=296, 400-4=396, 500-4=496. What do you notice? What would the next calculation be? Spot the mistake: 	

- Revise 3x table from Year 2 before teaching 4x table. Also, revise x9, x11, x12 strategies for all tables so far using a counting stick.
- Notes on inverse operations can be found on pages 11-14.

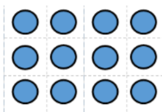
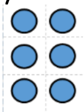
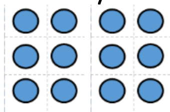
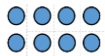
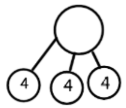
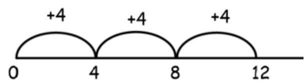
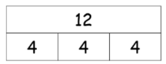
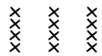
<u>Investigating the Distributive and Associative Law/Working with Larger Numbers (</u>These activities can take place with each table once the concept is secure, to deepen understanding of the properties and application of multiplication).

Notes	Concrete	Visual to Support Abstract
Begin to investigate the distributive (4x5+4x5=8x5) and associative (2x4x5) properties of multiplication informally during practical work.	In pairs, make an array of 4x10. Think about the different ways it can be put into equal groups and how to record.  E.g. Some may find 2x20, 4x10, 8x5. Can they split these further into 2x20 which is the same as 2 lots of 2 tens; or 4x10 which is the same as 4 lots of 2 fives etc. This could be explained as 2x10+2x10 or 4x2x5. Children may record in words (e.g. 2 lots of 2 tens or 2 tens + 2 tens) or just discuss this to explore the concept.	Draw the arrays and circle the different groups you have found. Write calculations. Draw arrays and number lines to represent calculations based around the distributive law, e.g. 3x3+2x4. Can they make up a problem it could represent? When confident, begin to think about how to visually represent the associative law e.g. 2x5x2 (or 2 lots of 5x2).
Use previous strategies and visuals to help understand multiplication and division with larger numbers and fractions. When working with hundreds, also refer to them as the number of tens. E.g. 300 or 30 tens (<u>30</u>0).	Explore the effect of multiplying by 10 on part whole/bar models. The bar model shows 10x10=100. Use equipment to find 20x10.  Predict 30x10, 40x10 etc. What do you notice? What about 11x10, 23x10 etc. Use tens counters (or imagine the ones counters are tens) to make the link between known facts and new facts. I know 3x4=12, so what else do I know? 30x4,4x30, 40x3, 3x40. Find associated division facts. Represent these new facts using apparatus that links back to previous work and known facts each time.    Explore with cuisenaire rods/number pieces. If each number piece is equivalent to one whole, how many quarters in 1? 2? 3? Mark a counting stick in ones and together mark on quarters. How many quarters will be in 5 whole ones?What do you notice?	Explore place value relationships on number lines and link back to single digits.    Bertie thinks he can show both the bar models above with an array made out of tens place value counters. Is he right? <u>Link to fractions: </u>Make the link between quarters and dividing by four explicit. E.g. Use cubes with bar models to quarter whole numbers by halving and halving again. Give one quarter on a bar model and use it to find the whole amount (double and double again). Also, reinforce this strategy when working with other fractions where four parts are needed e.g. 4/5,4/8 of a quantity. 

4x Table: Relationships - Skip Counting

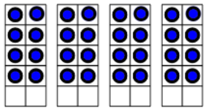
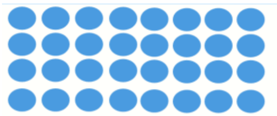
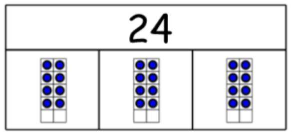
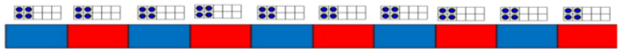
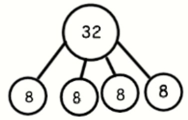
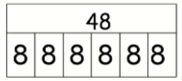
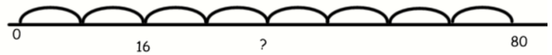
Notes	Concrete	Visual to Support Abstract
Revise repeated addition so secure. Show calculations and ask what they could mean, e.g. 3x4 could be 4 lots of 3 or 4 three times, 3 lots of 4 or 3 four times, 3 multiples of 4, 4 multiples of 3, 3+3+3+3 or 4+4+4. Ensure examples which include multiplying by 1 and 0 are included.	Represent and solve problems involving multiplication of four using different types of concrete apparatus. Match representations to calculations and vice versa.  4 + 4 + 4 + 4 = 4x4=  Use counting stick methods (see page 7) to explore the relationships between multiples of 4. Ensure discussion of 11x4 and 12x4 is included and focus upon x9 and x8 particularly, discussing whether this strategy would work for other multiples. E.g. Could you use it for 9x8? (E.g. 80-8). Write down numbers below 40. Which are multiples of 4? Predict which definitely cannot be multiples of 4. Explain why. Prove it. Combine apparatus into part whole models/bar models. Explore which are the parts and which is the whole. How could we say this? (5 lots of 4 or 4 five times).  Match apparatus to multiplication and division calculations and vice versa. Use them to solve missing number calculations practically. E.g. 20÷__=4. Use concrete apparatus to solve ____x___=40. What could it be? What couldn't it be? Revise link between tens and fives (e.g. 4x10 so how many fives?)	Draw empty number lines to solve problems.  Colour the pattern of fours on a 100 square. Discuss. What do you notice? Sort numbers. Which cannot be multiples of 4? How do you know?   Write the multiplication and division statements shown by the completed visuals. Use visuals to find missing numbers and solve problems. E.g. __x4=28. __÷4=9 Write a problem that the visual could represent.  If this shows 5x4, what would 10x4 be?  What tables fact does this show? How can it help you find 4x4? ___x___=40. What could it be? What couldn't it be? Use what is known about multiples of 1, 2,3, 4, 5, 10 to make predictions. Encourage systematic work.

4x Table: Investigating Relationships

Notes	Concrete	Visual to Support Abstract
These strategies rely heavily on the ability to double so this must be secure. See page 21 for introducing arrays. It is important to see the link between x4 and doubling early, as doubling is more efficient than skip counting, especially for later work with larger numbers or decimals. Include problems which involve scaling contexts, e.g. 4 times greater, 4 times as many/much and draw visuals to show these.	 What calculation does the array show? Asif says this shows 3x4=12 but Alex says it shows 4x3=12. Who is right? Explain. Can you use this array to help you work out 6x4? What do you think 12x4 would be? Play a game: show an array and use it to predict new tables facts. Link 2x table to 4x table. Make me an array to show 2x3. What do you think 4x3 would be? Why? Make an array to check.   Make or draw arrays to show different multiplication calculations from 4x table. What facts from 2x table would they show? What do you notice? Winston says any multiple of two will also be a multiple of four. Is he right? How could you find out? Discuss the relationship with division when working and make links to division calculations and problems. (See page Inverse Relationships, pages 11-14). Sort given multiplication calculations according to whether they are correct or incorrect. Explain how you knew, (e.g. Explain why 6x4=23 couldn't be correct). Use this as an opportunity to revise previous tables.	Write calculations to match arrays and draw arrays to match calculations. Look at word problems and find the appropriate multiplication calculation to represent it. Make up a different word problem for the same calculation. Draw arrays/bar models/number lines to show facts from the twos times table. Use them to predict facts from the 4x table.  E.g. 2x4=8, so 4x4=16. On a multiplication grid, record the first 5 facts from the 2 times tables and then repeat for the 4 times table. What do you notice? Represent multiplication and repeated addition statements visually, e.g. show 3 x 4 in four different ways. Write multiplication and division calculations to match and match visuals to calculations. Draw visuals to solve missing problems. E.g. 3x4. _÷4=6.    

		When showing tables facts, ask for all the different interpretations of what they could mean. E.g. 7x4 could mean 7 multiples of 4, 7 lots of 4, 7 four times, 4 lots of 7, 4 multiples of 7, 4 seven times etc. Discuss different strategies to work it out, drawing on strategies already known to reduce memory load and access a wider range of facts. E.g. 4x7 could be solved by thinking of it as four sevens and using doubling skills. $\underbrace{\underbrace{7+7}_{14}+\underbrace{7+7}_{14}}_{28}$
Reason to address misconceptions or help make generalisations.	• True or False: 9x4> 4x3. Explain how you know. • 5x4 < ____ What could go there? What couldn't? Why. • True or false: I can find the product of 5 and 4 by getting 4 lots of 5? • Show 7 lots of 4 on a number line/bar model/part whole model. Now show 4 lots of 7. What's the same? What's different? • Sophie says, "2x2x2 = 6" but Bartosz says she is wrong because "2x2x2=8" who is right. Prove it with concrete apparatus. Prove it with an array.	

8x Table: Relationships - Skip Counting

Notes	Concrete	Visual to Support Abstract
Revise and make clear the links to prior learning. Children should already know all facts from 0x8 to 5x8 from previous work and be able to quickly find x9, x11, x12. This should be discussed and made explicit. The only new facts are 6x8, 7x8,8x8. Ensure examples which include multiplying by 1 and 0 are included.	Use different types of concrete equipment to solve problems and represent repeated addition, multiplication and division calculations. Write calculations to match concrete representations.  8+8+8+8 4x8  Use cubes/counters to make arrays in eights. Write the multiplication and division statements to go with them. Read out problems and in pairs make the multiplication array to represent it.  Combine concrete equipment into part whole models and bar models. Explore which are the parts and which is the whole. Match calculations to these. Give missing number calculations and solve practically. E.g. __x8=24. Place fours (number pieces/arrangements of spots etc) on a counting stick or number line. Look at the fours and use to predict x8. Find 3x4, what will 3x8=?  Use counting stick methods (see page 7) to explore relationships between multiples of 8 and find more efficient ways to derive them using underlying skills and known facts. Ensure 11x8 and 12x8 is included. If they know 10x8, what would 20x8=? Make sure the relationship between x9 and x8 and the tenth multiple is focused upon particularly and discuss whether this strategy would work for other multiples. E.g. Could you use it for 9x8? (E.g. 80-8) 8x9? (90-16).	Solve problems and represent multiplication and division calculations by drawing arrays, number lines, part whole and bar models. Write calculations or problems to match visuals.   Use number relationships to fill in missing multiples in a number line. Explain how you found them.  Use number lines and counting sticks to explore how many different ways you can find tables facts. E.g. 8x8 could be (10x8)-16, double 4x8 etc. Use bar models/part whole models/number lines to find missing numbers in multiplication and division. E.g. __x8=32 48÷__=8. Always, sometimes, never? A multiple of 4, is also a multiple of 8. A multiple of 8 is a multiple of 4.

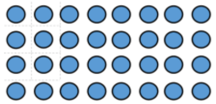
8x Table: Investigating Relationships

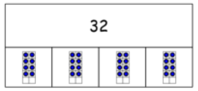
These strategies rely heavily on the ability to double so this must be secure.

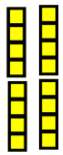
See page 21 for introducing arrays.

It is important to show a range of strategies for x8 so children begin to see the connections that exist within multiplication and have a number of strategies at their fingertips to access and apply facts quickly. Discuss the idea of making an amount 8 times greater. What would this look like on a number line? A bar model etc?

Match repeated addition and multiplication calculations to concrete and visual arrays.





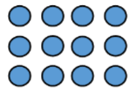


Link 4x table to 8x. Make me an array to show 4x8. What do you think 8x8 would be? Why? Make an array to check.

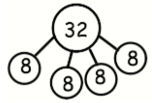
How can this bar model help you to find 8x8?
Look at equipment such as Cuisenaire rods, cubes etc. How many eights? How many fours? Write multiplication facts to match. What would the division fact be?
True or false: 3x8= 6x4? Show me how.
How many fours do you think would be in 9 eights?
Play a game. Show eights practically and ask children to predict on whiteboards the equivalent number of fours.

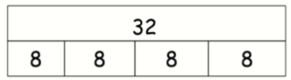
How many ways can you find 8x9 (Examples may be;
Count in 8s, find 4x9 and double it, find 10x8 and take 8 off, find 10x9 and take double 8 off). Examples could be given for children to prove with practical equipment.
Make an array to show 8 x 8. True or false: it also shows 2 lots of 4x8? What about 4 lots of lots of 2x2? What about 4x8+4x8?
Can you explore to find other ways to break the array down into smaller equal groups? If there are 4 lots of 8, how many lots of 16 will there be?

Look at this array. What multiplication and division facts from the 4x table does it show? Can you use it to draw the fact for the 8x table?



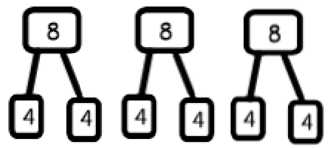
Colour in multiples of 4 and 8 in two different colours on a hundred square. What do you notice?
Fill in 4x table on multiplication grid. How could this help you find x8 facts?





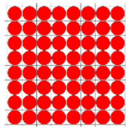
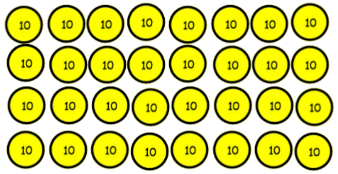
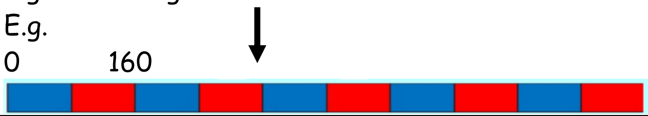
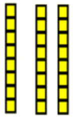
How could these help you find 8x8?

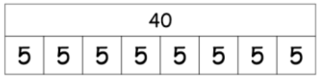
3x8=__x4? Prove it (bar model etc).



Which of these facts could this visual show: 6x4; 3x8; (2x4)+(4x4); 3x2x4? Show visuals and explore the different calculations one visual could represent.

Draw something to show your strategy to find 8x9.
Now draw a different way.

Using visuals to break numbers down can help explore the distributive and associative properties of multiplication in an informal way. Understanding of place value can be developed by using concrete apparatus to link the strategies used with single digit numbers to larger numbers.	 Concrete materials and visuals can be used to develop understanding of multiplication with larger numbers, by making the links to single digit numbers explicit. Make an array with ones. Now make the same array with tens (or imagine the ones are tens).  8x40=320 40x8=320 320÷8=40 320÷40=8 OR 80x4=320 320 ÷4=80 4x80=320 320÷80=4 A counting stick can also be used to make links between single digits and larger numbers. E.g. 0 160 	Draw the arrays made and circle the different groups found. Begin to explore how it might be represented in different ways. Could we show it on a bar model? If children are not secure with place value, drawing arrays where each circle/cross represents a ten instead of a one, can help consolidate their understanding. When showing the division link with larger numbers and practical equipment, sometimes it is appropriate to show it as sharing (e.g. 320÷4=80) and sometimes as grouping (320÷80=4).
Use knowledge of doubles and halves to link quarters and eighths.	 Use a counting stick or number equipment showing eighths. How many eighths are in 1 whole? 2? 3? etc. What do you notice? Find Cuisenaire rods to represent eighths and quarters. Investigate equivalence relationships between quarters and eighths and write calculations. E.g. 2 quarters = __ eighths.	<u>Link to fractions:</u> Draw bar models and other pictures divided into eighths to find out many eighths are in whole numbers. Link quarters and eighths. How many quarters would 6 eighths make? 4 eighths make? etc. What do you notice? Show it on bar model/fraction wall.

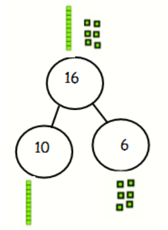
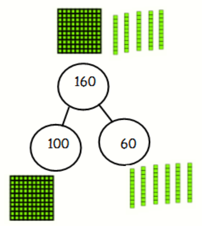
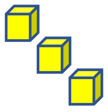
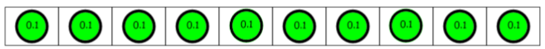
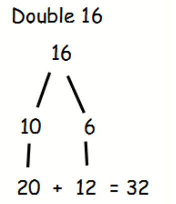
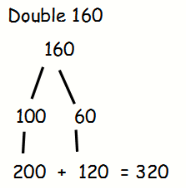
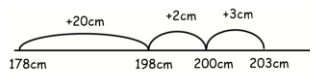
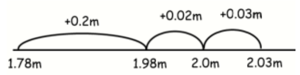
Reasoning to address misconceptions or help make generalisations.	• Which is the odd one out: 4+4+4+4+4+4+4+4 8+8+8+8 3+3+3+3 Why? • What's the same? What's different? 8+8+8 3+3+3+3+3+3+3+3 • True or false: 5x4<4x8. Explain. 2x4=8. 3x4=12. 4x4=16. What do you notice? What will 4x8=? 2x8=16. 3x8=24.  Louise says this represents 5 lots of 8. I she right? What multiplication facts does it show? What division facts? Can you write a problem that the bar model might represent?

Year 4 Guidance

Curriculum Links

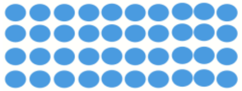
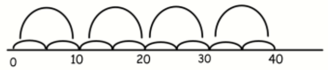
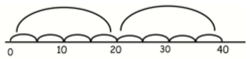
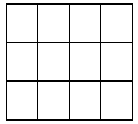
Underlying Skill	Linked to Y4 Learning Objectives
Doubling (near doubles), halving, bridging.	• Identify, represent and estimate numbers using different representations. • Add and subtract numbers with up to 4 digits using the formal written methods where appropriate. • Estimate and use inverse operations to check answers to a calculation. • Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why. • Solve number and practical problems that involve increasingly large positive numbers.
Multiplication/ Investigating Relationships.	• Identify, represent and estimate numbers using different representations. • Count in multiples of 6, 7, 9, 25 and 100. • Recall and use multiplication and division facts for the 6,7,9, 11 and 12 multiplication tables • Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers. • Recognise and use factor pairs and commutativity in mental calculations. • Multiply two-digit and three-digit numbers by a one-digit number using formal written layout. • Solve problems involving multiplying and adding including using the distributive law to multiply two-digit numbers by one digit • Solve problems involving multiplying and adding integer scaling problems and harder correspondence problems such as n objects are connected to m objects • Estimate and use inverse operations to check answers to a calculation. • Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as units, tenths and hundredths. • Estimate, compare and calculate different measures, including money in pounds and pence. • Convert between different units of measure (e.g. km to m; hour to minute). • Solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days. • Measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres. • Find the area of rectilinear shapes by counting squares. • Relationships also used throughout fraction work.

Underlying Skills: Doubles and Halves, Bridging, Subtraction from a Multiple of 10,100,1000.

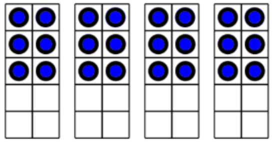
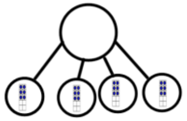
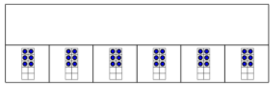
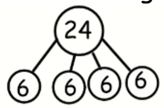
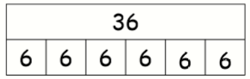
Notes	Concrete	Visual to Support Abstract
Build in opportunities to double, halve and bridge single digit numbers in formal written methods. Revise these skills with partitioning and place value in mental calculations with larger whole numbers and decimals in the context of money, measures and time. Revise and consolidate the importance of doubling and halving within the tables (i.e. between 5 and 10, 2 and 4, 4 and 8, 3 and 6) during work on fractions. E.g. Draw bar models/fraction walls linking fifths and tenths, thirds and sixths etc. Investigate whether finding one tenth can help to find one fifth or one third can help you to find a sixth. How could this help with division facts?	Use place value counters, Dienes, Numicon etc to represent and solve problems. Include doubling, halving, bridging during work with money using coins practically. (See Y2, page 30, if practical work needed on bridging).  E.g. doubling. 2x16=2x10, 2x6 so double 160=double 100, double 60.  Fractions link: Use Cuisenaire rods/cubes to revise doubling and halving with equivalent fractions. Make all the fractions you can that show one half. On a counting stick or number line marked in ones, show 5. How many halves will there be? What do you notice? Use cuisenaire rods (or cubes) to represent tenths in a whole amount. How many fifths will there be?  E.g. This is 3/10. Make the whole and put it in a bar model. How could you find out how many fifths is it equivalent to? Decimals link:  Share out between 2 people. They get 5/10 each which is the same as half. How will you write half as decimal? So how would one fifth be represented as a decimal? Show it on a bar model.	Draw place value counters or Dienes to help solve initially, moving towards a mental method.   E.g. Kemi's room is only 1.78 m wide, but Kyra's is 25cm longer. How long is Kyra's room?   203cm=2.03m. Fractions links: Write down all the fractions equivalent to one half. What do you notice? How many halves are in 2? 3? 4? etc. What do you notice? Why does this happen? Also, use doubles and halves links to explore other equivalent fractions. How many tenths are in one fifth? two fifths etc? How many sixths are in two thirds etc? Draw bar models/fraction walls to prove it.

- Guidance on inverse operations can be found on pages 11-14.
- The Distributive Property is included as part of work on each table in Year Four.

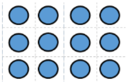
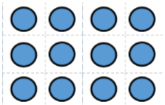
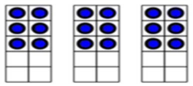
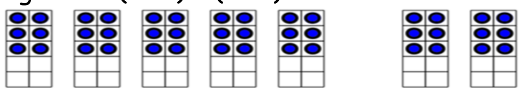
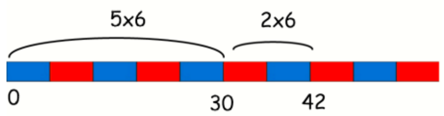
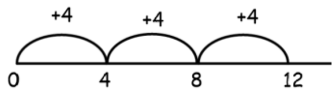
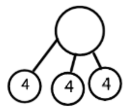
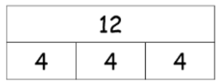
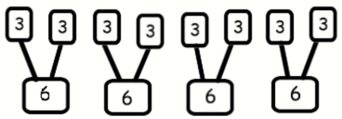
Multiplication: The Associative Property

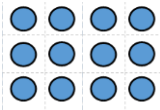
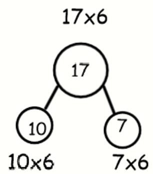
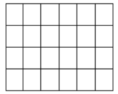
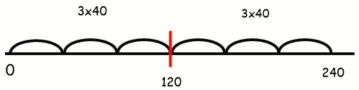
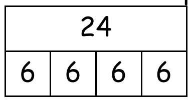
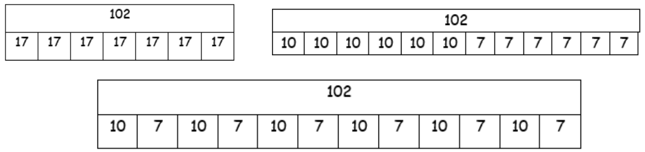
Notes	Concrete	Visual to Support Abstract
Investigate and talk about the associative property (i.e. 8x6=2x4x6 etc) practically. Use all known tables, beginning with those the children are likely to know well and moving on to new tables.	Make an array to show 10x4. True or false: it also shows 2 lots of 2x20 or 2 lots of 4x5?  Explore ways of breaking down arrays into smaller groups, beginning with familiar tables facts, then building onto new tables taught. Use multi-link to make cuboids to explore the associative property. E.g. make a cuboid to show 6x2x3. Build one face, then predict how many cubes you will need. Make it to find out. Can you make any other cuboids with the same number of cubes? Write the calculation that they show. Ravi is going to build a cuboid with sides measuring 4x5x3, whilst Ralph will build one with sides of 5x5x3. Who will need the most cubes to build it? How do you know? Use cubes to show how 2x30=20x3. (2x10x3). Extension: My cuboid is made of 40 cubes. How long could the sides be? Use cubes to find out. Could your multiplication and division facts help you work it out? Could one of the sides be 3 cubes long? How do you know? Show different cuboids (give dimensions) and ask the children to quickly work out on whiteboards its volume (by multiplying in the most efficient way). Discuss the different ways it was calculated.	Teacher could discuss ideas about what this might look like on a number line and bar model and model ideas about grouping. E.g. 40 is broken down into 4 lots of 2x5 (4x2x5).  Or 2 lots of 4x5 (2x4x5).  Draw circles round arrays to match the number lines. Match visuals to calculations and vice versa. In pairs, read out calculations from cards for partner to draw (e.g. 2 lots of 3x4 or 2x3x4). Match the calculations to the visuals drawn together and explain how you made your choice. Build cubes to represent different calculations. E.g. Build a cuboid to show this: 3x2x4. This is one face of my cuboid. Its total volume is less than 60 cubes.  What length could its other side be? What could it definitely not be?

6x Table: Relationships - Skip Counting

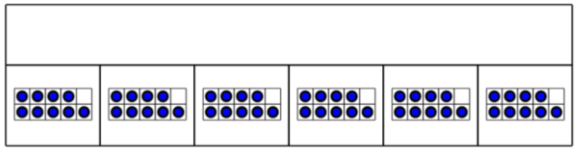
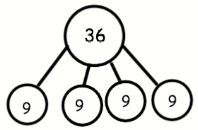
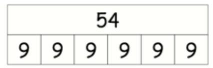
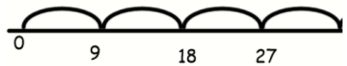
Notes	Concrete	Visual to Support Abstract
Ensure doubling, halving and bridging strategies are secure. Make explicit that most of x6 facts are already known from previous tables and revise these using the range of strategies already known. Only 6x6 and 6x7 are not known from another table or strategy. Ensure examples which include multiplying by 1 and 0 are included.	Represent and solve problems using concrete apparatus initially to show relationships.  6 + 6 + 6 + 6 = 4x6=24 24÷6=4 Use counting stick methods (see page 7) to explore relationships between multiples of 6 and find efficient ways to derive them using underlying skills and known facts. Ensure the link to 9x, 11x and 12x is made through use of the tenth multiple.  Make part whole models or bar models to represent calculations and problems. How many parts are needed? How many should there be in each part? What is the whole? Also, include missing numbers in different combinations.  Use to explore relationships. E.g. If this shows 6x6, what do you think 12x6 will be? Fractions link:  Use Cuisenaire rods or other practical equipment. If each piece represents one whole, how many sixths are in 1? 2? 3? etc. What do you notice? Why does this happen? Write down all the fractions equivalent to one sixth. What do notice? Mark a counting stick/number line in ones. How many sixths in 1? 2? 3? etc. Do you spot a pattern?	Colour the multiples of 6 on 100 square. Circle multiples of 3. What do you notice? Give out 2 digit numbers below 60. Which are definitely multiples of 6? Which can't be multiples of 6. How do you know? (E.g. odd numbers). Are there any facts that could help you quickly find out/check (e.g. link to multiples of other tables, such as threes or twelves). Represent multiplication and division calculations visually in different ways and use them to solve problems and find missing numbers. E.g. 6x__=24; 36÷__=6.   Write multiplication and division calculations represented by each visual. Explore multiples rules through problems such as: 60 ÷ __=__ Encourage children to work systematically. What definitely could go there? What definitely couldn't?
Reason to address misconceptions/generalise.	True or False: 9x>6 6x4. Explain how you know. 5x6 < ____ What could go there? What couldn't? Why. Sometimes, always never? Multiples of 3 are also multiples of 6. Show 100 square with some multiples of 5 and 6 circled. Spot which shouldn't be circled/are not multiples of 6.	

6x Table: Investigating Relationships

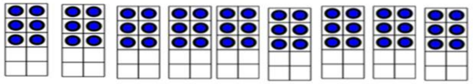
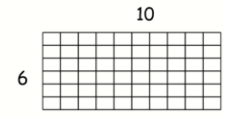
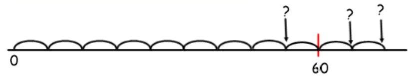
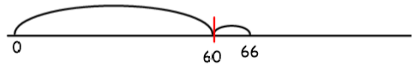
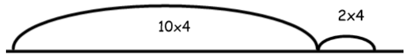
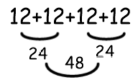
Notes	Concrete	Visual to Support Abstract
Make links between x6 and x3 and also between x6 and x5. See page 21 for introducing arrays. Include problems which involve making an amount 6x times greater and discuss how to represent. Explore distributive property of multiplication.	 Make an array to show 3x4. Write/match multiplication and division calculations to different arrays made. Does the array show 3x4=12 or 4x3=12? Explain. Investigate the relationships between 3 times table and 6 times tables. Show a number tripled on a counting stick and use it to predict the 6x fact.  Make me an array to show 3x4. What do you think 6x4 would be? Why? Prove it.  Use a range of equipment showing sixes (e.g. dice, Cuisenaire rods etc) to show 3x3. What would 3x6 be? Try with different amounts of threes. What do you notice? Now show sixes. So how many threes?  Use sixes to write facts from 6x table. Work out what the 3x facts would be. What do you notice? Use practical equipment, arrays and the counting stick to investigate the distributive property. E.g. 7x6= (5x6) + (2x6).  What else could this show? E.g. (2x6)+(3x6)+(2x6). 	Draw arrays (circles/crosses or rectangles drawn on squared paper) to show the facts from 6x table. Write 2 multiplication and 2 division calculations to match each array. (See page Inverse Relationships, page 12-15). Look at pictures of different visuals showing 3x table. Use them to find the related fact from 6x table. E.g.3x4=12, so 6x4=24.   xxxx xxxx xxxx Use visuals to find missing number facts (multiplication and division). True or false: 8x3=4x6. Explain why. Draw something to prove it.  Explore different calculations that visuals can show. E.g. Above could show: (3x3)+ (3x3) + (2x3) or (2x3) +(6x3) or (2x6)+(2x6). Draw visuals to show calculations shown in different ways, e.g. 5x6+3x6. Write an equivalent calculation.

	 If this is 4x3 what would 4x6 be? What does this array show? What if each counter was swapped for a tens counter? What could the array show now? If 3x40 =120, what does 6x40 =? Write all the multiplication and division facts if the counters are worth one. Repeat, imagining the counters are tens. What if each counter repesented 100? 10p? The distributive property should be explored an described when working practically and used in the context of real life problems. The distributive property can be used when working with 2 digit numbers and can support the understanding of formal written methods. Concrete apparatus can be used in part-whole models and bar models, moving onto representing visually.  True or false: 17x6=10x6+10x7?	Draw an array on squared paper to show 4x6.  Imagine each square is worth 10, what facts could the array show now? Visuals can be used to demonstrate and explain the distributive property and make links between known facts and larger numbers.  Explore the different calculations that the bar model below could represent. E.g. 2x6+2x6 or 1x6+3x6 etc.  Draw bar models, part whole models, number lines and arrays to show a range of calculations and problems. E.g. Jen had 3 boxes of 6 sweets and her friend gave her another 2 boxes of 6 sweets. How many sweets did she have altogether? 3x6+2x6. Which bar models could represent 17x6: 
Reason to address misconceptions or help make generalisations.	• True or false: I can find the product of 5 and 4 by getting 4 lots of 5? • Show 6 lots of 4 on a number line/bar model/part whole model. Now show 4 lots of 6. What's the same? What's different? • Sophie says, "2x3x4 must be greater than 4x3x2. Is she right? Explain how you know. Draw something to prove it.	

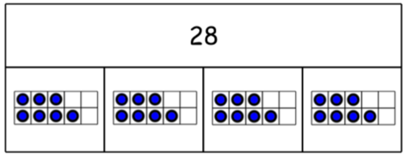
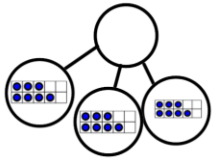
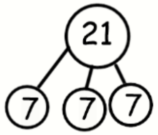
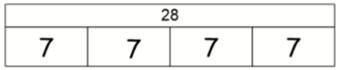
9x,11x,12x Table: Relationships - Skip Counting

Notes	Concrete	Visual to Support Abstract
Make explicit that most of x9, x11 and x12 facts are already known or can be quickly worked out using the tenth multiple. Practise using the 10th multiple before exploring skip counting. Ensure examples which include multiplying by 1 and 0 are included.	Use counting stick methods (see page 7) to explore relationships between multiples of 9 and find efficient ways to derive them using underlying skills and known facts. (Repeat when working with 11x and 12x). Use a hundred square and circle any multiple of nine. Get a stick of ten cubes and add them. What do you notice? Predict where the next multiple will be? Is there an easy way to find it? (Add ten and subtract one). How could you show this on a number line? Would this strategy be useful for adding 8? How do you think it could help when adding 11 or 12? Use apparatus showing nines to represent and solve problems and missing number calculations using part whole models and bar models for each table (9x,11x,12x). Explore which are the parts (how many parts, how many in each part) and which is the whole in calculations and problems.  Investigate flexible strategies when adding nine, such as swapping the ones to make an easier calcualtion. E.g. 27+9=20+7+9, so think of it as 29+7 then imagine the one moving over so it becomes 30+6.	Record the multiples of nine when working with a hundred square. What do you notice? What is happening to the tens digit each time. What is happening to the ones digit? Why? Repeat for 11x and 12x. True or false: if a number is a multiple of 9, its digits will always equal 9? Does this work with any multiple of 9? What about multiples of other numbers? Write repeated addition, multiplication and division calculations to match arrays, number lines, part whole models and bar models. Draw visuals to represent and solve calculations and missing number problems. E.g. 36÷ ___=9   True or false: The part whole model shows 2x9+2x9. What else could it show?

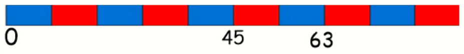
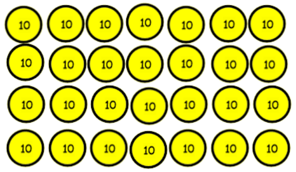
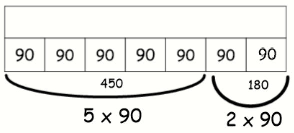
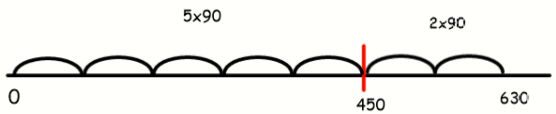
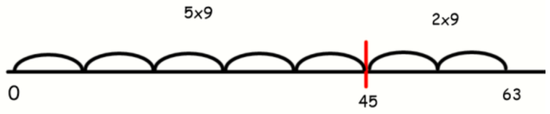
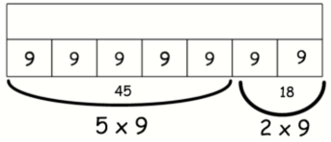
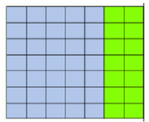
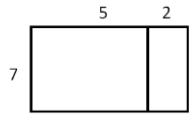
9x, 11x, 12x Table: Investigating Relationships

Notes	Concrete	Visual to Support Abstract
It is important to see the link between x10 and x9 (or 11x, 12x) and make use of the commutative property as this is generally the most efficient method. Also, investigate other relationships to give a range of methods to access different facts.	Revise the facts from 9x that are already known. Point to the tenth multiple on counting stick, number line and use practical apparatus. Ask children to work out 9x for all tables covered so far. (Repeat with 11x,12x). What tables fact are shown below? How can they help us find 9x6?   E.g. If this is 60, what multiple are we counting in? What would 9x6 be? How do you know? How about 11x,12x? How about if the tenth multiple is 50? What would 9x be? etc. (Repeat ideas when working with 11x table and 12x table). If 10x9=90, what would 20x9=? Could this help you find 19x? Investigate the relationship between the threes and nines using equipment such as Cuisenaire rods, Numicon etc  If this is 3x3, what would 6x3 look like? What about 9x3? Explain what you notice? Match repeated addition, multiplication and division calculations to the apparatus. Use apparatus to represent different calculations and to solve problems. Investigate the relationship between the sixes and twelves. Put out apparatus for 6x table. Show arrays or number line visuals from 3x or 6x table. Use them to find facts from 12x table.	Draw number lines and arrays to show 10x fact from different tables and use it to find 9x. (Repeat when working with 11x,12x).  How can we use this to find 9x6?  Moving towards:   Draw rectangles to show arrays from the 9x table. Give out cards with 2-digit numbers on. Which will be multiples of 9? Draw rectangles on squared paper to make arrays and find out. What do you notice about all the multiples of 9? Fill out 3x, 6x and 9x facts on a multiplication grid. What do you notice? Fill out the 6x and 12x times facts. What do you notice? What would happen if we doubled the 12x fact? Look at a calculation. What could it mean? How many ways can you find to solve it? Draw the two ways you find easiest. E.g. 12x4  

7x Table: Relationships - Skip Counting

Notes	Concrete	Visual to Support Abstract
Compare repeated addition and multiplication calculations. Ensure examples which include multiplying by 1 and 0 are included.	Use concrete apparatus in sevens to represent and solve problems. Match repeated addition, multiplication and division calculations to each other and to concrete representations. In what other ways could repeated additions be written? E.g. 7+7+7+7+7 + 7+7 could be written as 5x7 + 2x7. Match calculations to bar models and part whole models. Discuss which are the parts and which is the whole and use this help with solving missing number calculations and problems. E.g. ___x7=28. __÷7=3.    Use counting stick methods (see page 7) to explore relationships between multiples of 7 and to find efficient ways to derive them using underlying skills and known facts. Fractions link:  Use Cuisenaire rods which represent the seven (or other concrete equipment). How many sevenths in 1 piece? 2 pieces, 3 pieces etc. Mark a counting stick/number line in ones. How many sevenths in 1? 2? 3? etc. Do you spot a pattern?	Use representations to explore relationships and solve problems.  The part whole model shows 3x7. Use it to help you find 6x7.  Use the bar model to help you find 8x7. Draw a bar model or number line to show 3x7+2x7. Use a number line to show the relationships from the counting stick and find missing numbers.  Use a bar model or a number line to find the answer: ___x7=63. Write a problem that the calculation could represent. Fractions link: Write down all the fractions equivalent to one seventh. What do notice? Sort fractions that are equivalent to one seventh and those that are not.

7x Table: Investigating Relationships.

Notes	Concrete	Visual to Support Abstract
Investigate commutative property. Find many different ways to solve same calculations. See page 21 for introducing arrays. Explore distributive property, as this can be helpful for children who can't use other methods to recall the x7 facts, e.g. use of (x5) + (x2) for x7.	Use practical equipment to highlight that most of the x7 facts are already known from other tables. Write multiplication and division calculations to match arrays. Continue to use a range of representations to illustrate the distributive property and show how x5 and x2 facts can be used to find x7. E.g. 7x9 = 5x9 and 2x9.   Concrete and visuals can be used in the same way to help with the application of tables facts with larger numbers.  4x70=280 280÷70=4 70x4=280 280÷4=70. True or false: This array could also show 40x7? 280÷7?  How can 6x7 help you find 60x7? Make/draw an array with tens counters to show me. I know 6x7=42. What else do I know?	Represent visually on number lines, bar models etc.  Draw rectangles to show arrays which represent facts in the 7x table. Show how this can split into the x5 fact and the double.  E.g. 7x7= 5x7 and 2x7= 35+14=49. As children become confident, blank arrays can also be drawn to save time. E.g.  True or false: 6x7>7x5. Explain your thinking. How many ways can you find to solve this tables fact: 7x8. What known facts and strategies could help you? E.g. Find 7x4 twice, find 5x8+2x8, find 10x7 an subtract two sevens. Visually represent the different ways of thinking about the calculation.

Year 5 and 6 Guidance

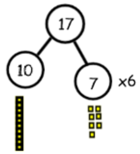
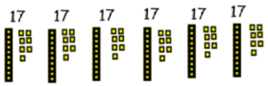
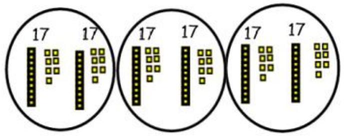
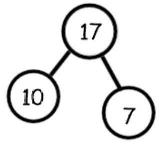
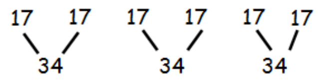
The curriculum assumes that all children enter Upper School with a firm understanding of the mental strategies needed to recall, manipulate and utilise their times tables fluently. In this section, we have included examples of how these earlier strategies may be applied and consolidated within the Year 5 and 6 curriculum and added examples of practical activities to support and reinforce these. The intention is that concrete materials expose the structure of the relationships and help give an understanding of the underlying concepts, whilst the visual helps children to internalise and visualise relationships so they gain the understanding to work in the abstract. For children who are not fluent, look for opportunities to revise earlier strategies.

- Guidance on inverse operations can be found on pages 11-15.
- It may also be useful to refer to the guidance for the Associative Property on page 43 and 53.

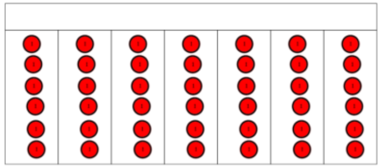
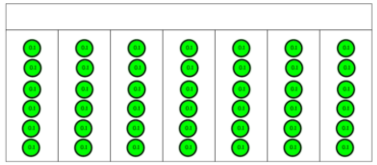
Opportunities to revise underlying skills

Objective	
Y5 Add and subtract numbers mentally with increasingly large numbers. Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction). Y6 Perform mental calculations, including with mixed operations and large numbers. Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why. Y6 Illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius. find unknown angles in any triangles, quadrilaterals and regular polygons.	• Consolidate strategies for doubling, halving, tripling and bridging from previous year groups whilst teaching these objectives. • Then use knowledge of place value, alongside practical apparatus, to apply for use with larger numbers, decimals and fractions.

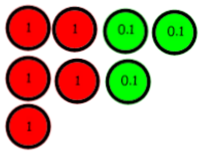
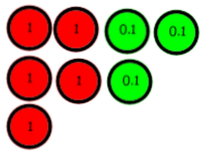
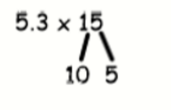
Using Tables Facts with Larger Numbers and Decimals

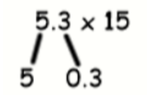
Linked Objectives	Notes	Concrete	Visual to Support Abstract
Y5 and 6 Multiply and divide numbers mentally drawing upon known facts Knowledge also required for use in long and short multiplication and division. Use knowledge of factors to multiply.	Use the properties of multiplication and known facts to work with larger numbers and decimals in same way as in previous years.	E.g. Explore different ways of finding 17x6. E.g. (10x6)+(7x6). 10x6=60 7x6=42  Consolidate understanding of how factors can be used to multiply, e.g. 17x6=17x2x3 (or 34x3). What would it look like drawn as 17x3x2?   34 x 3 What would 170x6 be? What about 1.7x6?	 (10x6)+(7x6)= 60+42=102  30x3=90 4x3=12 Visuals such as these can be used to gain understanding of the strategy of doubling one side and halving the other to gain an equivalent calculation. E.g. 17x6=34x3.
Count forwards or backwards in steps of powers of 10. (Y5) Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 (Y5).	Link larger numbers and decimals to known facts. Use to consolidate previous table knowledge and relationships. E.g. look at 3 on counting stick and tripling link.	Use the counting stick and post-it notes to put on the 1st, 10th and 5th multiples as markers. See section 'Using the Counting Stick', page 7). Explicitly link tables facts to new facts through place value relationships. Place value counters and sliders can be used alongside this. E.g. 0.4 0.8 1.2  40 80 120 This can also be used to explore the associative law. 7 x 40 = (7 x 4) x 10 = 280. Make links to known facts practically.	Draw a number line to represent the counting stick. Fill in missing numbers based on knowledge of relationships and place value, revising previous strategies and applying to larger numbers and decimals. 

<table>
<tr>
<td>Generate and describe linear sequences. (Y6)
Multiply and divide numbers by 10, 100 and 1000 (up to 3dp). (Y6).

Multiply one-digit numbers with up to two decimal places by whole numbers. (Y6).</td>
<td>When working with decimals, Dienes equipment can also be useful, with a hundred block representing one whole, tens representing tenths and ones representing hundredths.</td>
<td>7x6=42

7x0.6=4.2.
The array formation can also show 6x0.7.

If 7x0.6=4.2, what would 7x1.6 be?
If 7x0.6=4.2, what would 14x0.6= etc?

Use a place value grid/place value slider to show the effect of multiplying and dividing by 10, 100 etc alongside practical apparatus to support understanding of place value.

Once place value knowledge is secure, apply doubling and halving strategies from previous tables.
5.3x15






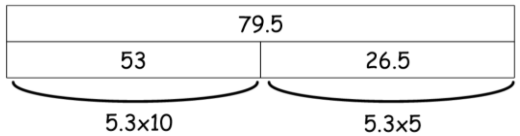
5.3x10=53
5.3x5=26.5
53+26.5=79.5.

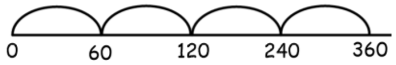
5x15=75.
0.3x15= 4.5.
75+4.5=79.5.</td>
<td>Draw place value counters alongside if needed to make links between known facts and decimals.

If 7x6=42, what would 7x60 be? What about 7x0.6? 7x6000?
Predict what will happen on place value slider or place value grid.
<table>
<tr><td>Th</td><td>H</td><td>T</td><td>O</td></tr>
<tr><td></td><td></td><td>2</td><td>8</td></tr>
<tr><td></td><td>2</td><td>8</td><td>0</td></tr>
</table>

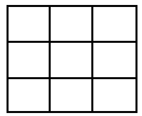
Draw arrays /rectangles where each square is worth 0.1/10,100,1000 etc to prove it.

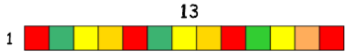
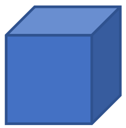
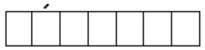
Write fact families that could be represented by the arrays (include division facts).
Use arrays, bar models and number lines to solve missing number problems, e.g.
____÷0.7=6.

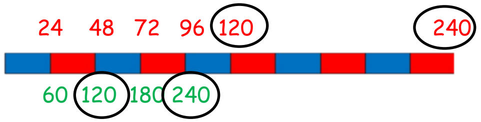
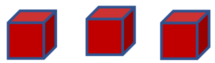
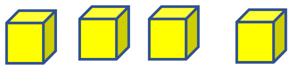
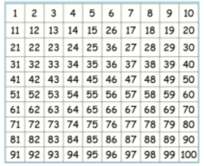
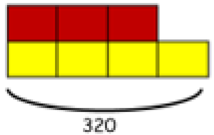
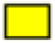
Draw place value counters alongside calculations, moving onto bar models to reveal relationships, before working in the abstract.
E.g. 5.3x15.

</td>
</tr>
</table>

Convert between different units of metric measure, including problems involving conversion of time. (Y5) Solve problems involving calculation and conversion of units of measure (length, mass, volume, time), using decimal notation (up to 3dp). (Y6).	Look for contexts which may lend themselves to consolidation of particular tables.	E.g. Use work with conversion of time to revise x6 table. E.g. Time: 6 12 18 24 etc  60 120 180 240	 Use a number line and knowledge of 6x table to convert between units of time, e.g. Convert 265 minutes into hours.

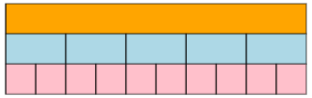
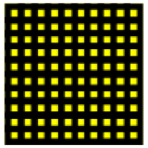
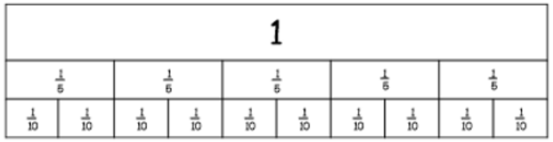
Exploring Tables Facts through Common Multiples and Factors, Prime, Square and Cube Numbers.

Curriculum Links	**Notes**	**Concrete**	**Visual to Support Abstract**
Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes (Y5) Recognise that shapes with the same areas can have different	Encourage children to predict and generalise as they are working with practical apparatus. Also, make links to larger numbers and decimals, What if each	 Use multi-link cubes to make as many different rectangles as you can out of a given numbers of cubes. Find factor pairs. Predict all of the factors for each number given. Write calculations to match. Predict based on doubles/halves relationships. E.g. If 10 is a factor, what else will be a factor? Are there any numbers that will only make one rectangle? Predict which numbers will only make a square (no other rectangles). Why are these called square numbers?	 Draw rectangles to represent practical work. Predict and generalise about relationships within multiplication. E.g. If a 3x8 rectangle was drawn, if you double one side, what happens to the other side? If 60÷6=10, what would 60÷3 be? What about 60÷12? Etc. What do you notice about the numbers that will only form a square? Can you find any others?

perimeters and vice versa (Y6) Establish whether a number up to 100 is prime and recall prime numbers up to 19. (Y5) Recognise and use square numbers and the notation for squared (^{2}) cube numbers and the notation for cubed (^{3}) (Y5). Calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units (e.g. mm and km). (Y6) Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers (Y5) Identify common factors, common multiples, prime numbers (Y6).	cube was worth 10, 100, 0.1 ?etc.	Use multi-link cubes or draw rectangles to systematically investigate all factors of a number. Record on factor rainbow. Find out which numbers from 1-20 can only make one rectangle made from a single line of cubes. E.g.  Consider the numbers up to 20. Which cannot be prime? Explain why. Get 16 multi-link cubes. How many different cuboids can you make? Write the multiplication sentences to go with each. Try with different numbers of cubes. Investigate which numbers can build a cube? Why is this?  Match/write different calculations to go with each cube/cuboid? Make cuboids of given dimensions e.g. 6x3x2. Show a cuboid and give its dimensions. Can the children quickly calculate its volume? Discuss the different ways it was solved. Make one face of a cube. Give to a partner. What cube number would it make? Build it to check. Explore the relationship between square and cube	A rectangle has an area of 48. What could its sides be? Represent as a factor rainbow.  What could its perimeter be?  Draw rectangles to identify which numbers only have one possible rectangle (where the cubes make one line). Sort numbers up to 20 according to whether or not they are prime. Which numbers do you know definitely can't be prime? Give out square numbers and prime numbers on cards and draw rectangles to represent them. Predict which will be prime? Which will be square? Draw to prove it. Draw cuboids and write/match calculations to go with them. Write equivalent calculations __x __ x__ = __x __ x__ Predict a calculation for another cube with the same volume. 3x4x5=__x2x6. Use cubes to check. Discuss different ways to solve. Use this to revise use of factors to find more efficient ways to multiply (i.e. associative law). E.g. 7x16= 7x8x2= 56x2 or 112.

Fractions link: Compare and order fractions whose denominators are all multiples of the same number. (Y5) Fractions link: Use common factors to simplify fractions. Use common multiples to express fractions in the same denomination (Y6) Add and subtract fractions with the same denominator and denominators that are multiples of the same number (Y6) Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and x3=division facts (Y6).	Finding common multiples involves doubling, tripling and bridging strategies from previous years. Look back to strategies if children not secure. Problems such as these can provide opportunities for halving (by using halving and halving again to divide by 4) and tripling strategies.	numbers. Show cubes for children to predict the volume. Use a counting stick to find common multiples using two different coloured sticky notes for the different times tables. E.g. Find the common multiples of 24 and 60.  Jessica thinks 32 is a common multiple of 8, 12 and 16. Is she right? Explain how you know. Use Cuisenaire rods or cubes with bar models to show ratio. E.g. For every £3 Aisha saves, Ben saves £4. Ben has saved up £320. How much has Aisha saved? Aisha  Ben 	 Use different coloured pens to highlight the indivdual times tables and then identify which ones they have in common to find common factors/multiples. Draw bar models to show/work out relationships.   =320 320 ÷ 4= 80  =80 £80x3=£240

Consolidation through Fractions, including Decimals and Percentages).

Curriculum Links	**Notes**	**Concrete**	**Visual to Support Abstract**
Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths. (Y5) Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions. (Y6) Compare and order fractions, including fractions >1. (Y6) Divide proper fractions by whole numbers (e.g. 1/3 ÷ 2 = 1/6. (Y6)	Make links between thirds and sixths, quarters and eighths, sixths and twelfths etc by using doubling and halving strategies.	Revise underlying relationships (e.g. doubles/halves) for each table when working with fractions. For example, when working with fifths, arrange Cuisenaire rods into a bar model then discuss how many tenths it is equivalent to. Discuss the link between x5 and x10 in whole numbers and tenths and fifths in fractions.  How many tenths? How many fifths? What do you notice? How many fifths in 2? How many tenths in 2? Use a counting stick as one whole. Mark on fifths, tenths etc and discuss relationships. If this is one whole, what would a tenth look like? What would a fifth look like?  Put Dienes or place value counters in bar models and compare relationships between fifths and tenths. Repeat to link quarters and eighths, thirds and sixths etc.	 Karen says that you can find one tenth by doubling one fifth? Is she right? Prove it. Colour one tenth on a 100 square if the 100 square represents one whole. Colour one fifth etc. Record what you colour as a decimal, as a percentage, as tenths/hundredths etc. Revise strategies for individual tables when working with fractions. E.g. If 3/8=21, what will 6/8 be? Write down all the fractions equivalent to 1/3. What do you notice? Repeat with other fractions. Which of the fractions equivalent to one third can you make equivalent to one sixth? Show it on a bar model. Predict those which you won't be able to convert to one sixth. Explain why.

Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements as a mixed number (E.g.: 2/5 + 4/5 = 6/5 = 1 1/5) (Y6)		How many eighths are in 4 whole ones? How can you find out? Use equipment such as Cuisenaire rods, Numicon or number lines/counting sticks to explore. Look to appropriate year group to reinforce tables strategy when working with given fractions. How many eighths in 2 whole ones? How can this help you work out how many in 4 whole ones?	Reinforce tables facts through work with mixed numbers. E.g. True or false: 15/4 > 23/8. Explain how you know.

<u>References:</u>

Boaler, J. (2015). Fluency Without Fear: Research Evidence on the Best way to Learn Maths Facts [online] [pdf.]

Available at:< https://www.youcubed.org/evidence/fluency-without-fear/>

Boaler, J., Munson, J., Williams C. (2017). Mindset Mathematics: Visualising and Investigating Big Ideas, Grade 4. Jossey-Bass. San Francisco, CA.

NATIONAL CENTRE FOR EXCELLENCE IN THE TEACHING OF MATHEMATICS. *Teaching for Mastery: Five Big Ideas.* [online]. July 2017.

Available at: <https://www.ncetm.org.uk/resources/50042>. (Accessed September 2017).

NATIONAL CENTRE FOR EXCELLENCE IN THE TEACHING OF MATHEMATICS. *Developing Mastery in Mathematics [pdf][online].* October 2014.

Available at:< https://www.ncetm.org.uk/public/files/19990433/Developing_mastery_in_mathematics_october_2014.pdf>

Association of Teachers of Maths, 2009. *Times Tables in Ten Minutes.* [online video]. Available at: <https://www.youtube.com/watch?v=yXdHGBfoqfw>

Youcubed at Stanford (2014). From Stanford Online's 'How To Learn Math for Teachers and Parents': Number Talks. [online video]. Available at:

< https://www.youtube.com/watch?v=yXNG6GKFhQM>